Preface

Sobolev spaces, i.e., the classes of functions with derivatives in L_p, occupy an outstanding place in analysis. During the last half-century a substantial contribution to the study of these spaces has been made; so now solutions to many important problems connected with them are known.

In the present monograph we consider various aspects of theory of Sobolev spaces in particular, the so-called embedding theorems. Such theorems, originally established by S.L. Sobolev in the 1930s, proved to be a useful tool in functional analysis and in the theory of linear and nonlinear partial differential equations.

A part of this book first appeared in German as three booklets of Teubner-Texte für Mathematik [552, 555]. In the Springer volume of "Sobolev Spaces" [556] published in 1985, the material was expanded and revised.

As the years passed the area became immensely vast and underwent important changes, so the main contents of the 1985 volume had the potential for further development, as shown by numerous references. Therefore, and since the volume became a bibliographical rarity, Springer-Verlag offered me the opportunity to prepare the second, updated edition of [556].

As in [556], the selection of topics was mainly influenced by my involvement in their study, so a considerable part of the text is a report on my work in the field. In comparison with [556], the present text is enhanced by more recent results. New comments and the significantly augmented list of references are intended to create a broader and modern view of the area. The book differs considerably from the monographs of other authors dealing with spaces of differentiable functions that were published in the last 50 years.

Each of the 18 chapters of the book is divided into sections and most of the sections consist of subsections. The sections and subsections are numbered by two and three numbers, respectively (3.1 is Sect. 1 in Chap. 3, 1.4.3 is Subsect. 3 in Sect. 4 in Chap. 1). Inside subsections we use an independent numbering of theorems, lemmas, propositions, corollaries, remarks, and so on. If a subsection contains only one theorem or lemma then this theorem or lemma has no number. In references to the material from another section or subsection we first indicate the number of this section or subsection. For example, Theorem 1.2.1/1 means Theorem 1 in Subsect. 1.2.1, (2.6.6) denotes formula (6) in Sect. 2.6.

The reader can obtain a general idea of the contents of the book from the Introduction. Most of the references to the literature are collected in the Comments. The list of notation is given at the end of the book.

The volume is addressed to students and researchers working in functional analysis and in the theory of partial differential operators. Prerequisites for reading this book are undergraduate courses in these subjects.

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Liverpool–Linköping Vladimir Maz'ya
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