# Preface

Infimum and supremum are indispensable concepts in optimization. Nevertheless their role in vector optimization has been rather marginal. This seems to be due the fact that their existence in partially ordered vector spaces is connected with restrictive assumptions. The key to an approach to vector optimization based on infimum and supremum is to consider set-valued objective functions and to extend the partial ordering of the original objective space to a suitable subspace of the power set. In this new space the infimum and supremum exist under the usual assumptions.

These ideas lead to a novel exposition of vector optimization. The reader is not only required to familiarize with several new concepts, but also a change of philosophy is suggested to those being acquainted with the classical approaches. The goal of this monograph is to cover the most important concepts and results on vector optimization and to convey the ideas, which can be used to derive corresponding variants of all the remaining results and concepts. This selection ranges from the general theory including solution concepts and duality theory, through to algorithms for the linear case.

Researchers and graduate-level students working in the field of vector optimization belong to the intended audience. In view of many facts and notions that are recalled, the book is also addressed to those who are not familiar with classical approaches to vector optimization. However, it should be taken into account that a fundamental motivation of vector optimization and applications are beyond the scope of this book.

Some basic knowledge in (scalar) optimization, convex analysis and general topology is necessary to understand the first part, which deals with general and convex problems. The second part is a self-contained exposition of the linear case. Infimum and supremum are not visible but present in the background. The connections to the first part are explained at several places, but they are not necessary to understand the results for the linear case. Some knowledge on (scalar) linear programming is required.

The results in this book arose from several research papers that have been published over the last five years. The results and ideas of this exposition

are contributed by Andreas Hamel, Frank Heyde and Christiane Tammer concerning the first part as well as Frank Heyde, Christiane Tammer and Matthias Ehrgott concerning the second part. A first summary, extension and consolidation of these results has been given in the author's habilitation thesis, which appeared in 2010. This book is an extension. It contains one new chapter with extended variants of algorithms and more detailed explanations.

I thank all persons who supported me to write this book. In particular, I'm greatly indebted to Matthias Ehrgott, Gabriele Eichfelder, Andreas Hamel, Frank Heyde, Johannes Jahn and Christiane Tammer for their valuable comments, important corrections and all their advice that entailed a considerable increase of quality.

Halle (Saale), *Andreas Löhne*
November 2010