Preface

Classical graph theory has been developed in regard to its applications in urban planning, transport, energetics, and many other fields. The general optimization mindset dominating these researches has addressed to graph theory the questions which were often related to finding the *shortest path* between nodes, as being of the minimum time delay for information transmission and of the minimum cost for connection maintenance. Not surprisingly, the very definition of distance between two vertices in a graph is given as the geodesic distance, i.e., the shortest path connecting them. With respect to the graph metric, a complex network of weighted edges is rather considered as a minimum weight spanning tree of the underlying graph, i.e., a subset of paths that has no cycles but still connects to every vertices at the lowest total cost. However, in many problems of practical interest found in everyday life the existence of many paths of different lengths as well as a nexus of cycles traversing the nodes in many complex networks do matter!

In contrast to classical graph theory paying attention to the shortest paths of least cost, in the approach that we discuss in our book, *all possible paths* between the two vertices in a connected graph are taken into account, although some paths shall be more preferable than others. Such a formulation of graph theory can be called as of a "*path integral*", since "integral" means "to include all." Random walks respecting all graph symmetries assign a probability to each path in the graph to be traversed by a random walker. Then, in order to find the expected first-passage distance between the two vertices, one integrates over all possible paths of the system in between them. Consequently, each vertex is characterized with respect to the entire structure of the graph by its own "path integral" vector accounting for the sum of the probability amplitudes for every possible path leading to that from a randomly chosen vertex. Perhaps, the most interesting fact about such a "path integral" approach to graphs is that the probabilistic distance naturally induces a Euclidean metric on a graph (sometimes called the 'diffusion metric', or the 'effective resistance metric') allowing for a geometric representation of the relationships between vertices in a graph, in terms of distances and angles, as in Euclidean geometry of everyday intuition. Vertexes of graphs and units of data bases that cast in the same mold with respect to the individual data features are revealed by geometric proximity in Euclidean space that might be either exploited visually, or accounted analytically. High-dimensional Euclidean representations of graphs and databases are characterized by the rank-ordering of data traits providing us with the natural geometric framework for dimensionality reduction facilitating the data analysis and further interpretation of results.

Perhaps, Lagrange was the first scientist who investigated a simple dynamical process (diffusion) in order to study the properties of a graph (Lagrange 1867). He calculated the spectrum of the Laplace operator defined on a chain (a linear graph) of N nodes in order to study the discretization of the acoustic equations. Nowadays it is well known that random walks could be used in order to investigate and characterize how effectively the nodes and edges of large networks can be covered by different strategies (see Tadic 2002; Yang 2005; Costa and Travieso 2007 and many others).

In this book, we follow the interdisciplinary lecture course on the stochastic analysis of complex networks and databases delivered by us at the University of Bielefeld (Germany) during the Fall semester 2008 and the Spring semester 2009 and targeted to bring about a more interdisciplinary approach across diverse fields of research including complex network theory and data analysis, as well as sociology, bio-informatics, urban planning and linguistics. The book contains a wealth of material generously equipped with suggestions for further reading and the glossary of term and concepts in graph theory that is helpful for those at the beginning of their acquaintance with the subject.

In the subsequent ten chapters of this book, we describe a fascinating journey from the elementary discrete mathematics (Chaps. 1, 2) to the elements of algebraic graph theory (Chap. 3), to a detailed analysis of complex multicomponent systems and databases (Chaps. 4–7), to the applications of random walk methods for the components analysis of complex networks and databases (Chap. 8). In the Chap. 9, we discuss the dynamical processes in models containing a large number of positive and negative feedbacks such as epidemic spreading, synchronization, and self-regulation in complex genetic networks. Finally, in the Chap. 10, we consider strongly non-linear transport phenomena in large complex networks containing regular subgraphs.

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