

Prefaces

Preface to the First Edition

Linear evolution equations in Banach spaces have seen important developments in the last two decades. This is due to the many different applications in the theory of partial differential equations, probability theory, mathematical physics, and other areas, and also to the development of new techniques. One important technique is given by the Laplace transform. It played an important role in the early development of semigroup theory, as can be seen in the pioneering monograph by Hille and Phillips [HP57]. But many new results and concepts have come from Laplace transform techniques in the last 15 years. In contrast to the classical theory, one particular feature of this method is that functions with values in a Banach space have to be considered.

The aim of this book is to present the theory of linear evolution equations in a systematic way by using the methods of vector-valued Laplace transforms.

It is simple to describe the basic idea relating these two subjects. Let A be a closed linear operator on a Banach space X . The *Cauchy problem* defined by A is the initial value problem

$$(CP) \quad \begin{cases} u'(t) = Au(t) & (t \geq 0), \\ u(0) = x, \end{cases}$$

where $x \in X$ is a given initial value. If u is an exponentially bounded, continuous function, then we may consider the Laplace transform

$$\hat{u}(\lambda) = \int_0^\infty e^{-\lambda t} u(t) dt$$

of u for large real λ . It turns out that u is a (mild) solution of (CP) if and only if

$$(\lambda - A)\hat{u}(\lambda) = x \quad (\lambda \text{ large}). \quad (1)$$

Thus, if λ is in the resolvent set of A , then

$$\hat{u}(\lambda) = (\lambda - A)^{-1}x. \quad (2)$$

Now it is a typical feature of concrete evolution equations that no explicit information on the solution is known and only in exceptional cases can the solution be given by a formula. On the other hand, in many cases much can be said about the resolvent of the given operator. The fact that the Laplace transform allows us to reduce the Cauchy problem (*CP*) to the characteristic equation (1) explains its usefulness. The Laplace transform is the link between solutions and resolvents, between Cauchy problems and spectral properties of operators.

There are two important themes in the theory of Laplace transforms. The first concerns representation theorems; i.e., results which give criteria to decide whether a given function is a Laplace transform. Clearly, in view of (2), such results, applied to the resolvent of an operator, give information on the solvability of the Cauchy problem.

The other important subject is asymptotic behaviour, where the most challenging and delicate results are *Tauberian theorems* which allow one to deduce asymptotic properties of a function from properties of its transform. Since in the case of solutions of (*CP*) the transform is given by the resolvent, such results may allow one to deduce results of asymptotic behaviour from spectral properties of A .

These two themes describe the essence of this book, which is divided into three parts. In the first, representation theorems for Laplace transforms are given, and corresponding to this, well-posedness of the Cauchy problem is studied. The second is a systematic study of asymptotic behaviour of Laplace transforms first of arbitrary functions, and then of solutions of (*CP*). The last part contains applications and illustrative examples. Each part is preceded by a detailed introduction where we describe the interplay between the diverse subjects and explain how the sections are related.

We have assumed that the reader is already familiar with the basic topics of functional analysis and the theory of bounded linear operators, Lebesgue integration and functions of a complex variable. We require some standard facts from Fourier analysis and slightly more advanced areas of functional analysis for which we give references in the text. There are also four appendices (A, B, C and E) which collect together background material on other standard topics for use in various places in the book, while Appendix D gives a proof of a technical result in the geometry of Banach spaces which is needed in Section 4.6.

Finally, a few words should be said about the realization of the book. The collaboration of the authors is based on two research activities: the common work of W. Arendt, M. Hieber and F. Neubrander on integrated semigroups and the work of W. Arendt and C. Batty on asymptotic behaviour of semigroups over many years. Laplace transform methods are common to both.

The actual contributions are as follows.

Part I: All four authors wrote this part.

Part II was written by W. Arendt and C. Batty.

Part III was written by W. Arendt (Chapters 6 and 7) and M. Hieber (Chapter 8).

C. Batty undertook the coordination needed to make the material into a consistent book.

The authors are grateful to many colleagues and friends with whom they had a fruitful cooperation, frequently over many years, which allowed them to discuss the material presented in the book. We would especially like to acknowledge among them H. Amann, B. Bäumer, Ph. Bénilan, J. van Casteren, R. Chill, O. El-Mennaoui, J. Goldstein, H. Kellermann, V. Keyantuo, R. deLaubenfels, G. Lumer, R. Nagel, J. van Neerven, J. Prüss, F. Rübiger, A. Rhandi, W. Ruess, Q.P. Vū, and L. Weis

Special thanks go to S. Bu, R. Chill, M. Haase, R. Nagel and R. Schnaubelt, who read parts of the manuscript and gave very useful comments.

The enormous technical work on the computer, in particular typing large parts of the manuscript and unifying 4 different TEX dialects, was done with high competence in a most reliable and efficient way by Mahamadi Warma. To him go our warmest thanks.

The authors are grateful to Professor H. Amann, editor of “Monographs in Mathematics”, for his support. The cooperation with Birkhäuser Verlag, and with Dr. T. Hintermann in particular, was most enjoyable and efficient.

Ulm, Oxford, Darmstadt, Baton Rouge
August, 2000

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Preface to the Second Edition

Ten years after the publication of the first edition of this monograph, it is clear that vector-valued Laplace transform methods continue to play an important role in the analysis of partial differential equations and other disciplines of analysis. Among the most notable new achievements of this period are the characterization of generators of cosine functions on Hilbert space due to Crouzeix, and quantitative Tauberian theorems for Laplace transforms with applications to energy estimates for wave equations.

In this second edition, the new developments have been taken into account by updating the Notes on each Chapter and the Bibliography. For example, the characterization of generators of cosine functions on Hilbert space by a purely geometric condition on the numerical range is precisely stated in Theorem 3.17.5. The main text has not been substantially changed, except in Section 4.4 where some results are now presented in quantitative forms. Their applications in the study of damped wave equations are explained in detail in the Notes of the section.

A few minor mathematical gaps and typographical errors have been corrected, and we are grateful to M. Haase, J. van Neerven, R. Schumann and D. Seifert for alerting us to some of them.

September 2010

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