Preface

Nanotechnology is a key technology of the 21st century. It investigates very small structures of the size of a few nanometers up to several 100 nanometers. Thus, these structures are often smaller then the wave length of light. In this book, we concentrate on the mathematics of photonic crystals, which form an important class of physical structures investigated in nanotechnology.

The investigation of these structures by mathematical methods is highly important for the following reasons.

- Since the physical behaviour in the nanoscale is very difficult and expensive to measure in real experiments, numerical simulations play a fundamental role in understanding such processes. In many cases theses structures are fully three dimensional, and process on very different scales in space and time, so that constructing efficient and reliable simulation methods is a true mathematical challenge.
- Often ill-posed problems arise in a natural way, e. g., in the reconstruction problem of nanostructures from measurements. Here, methods of the theory of inverse problems must be developed and applied.
- Often it is not possible, e. g. due to very large differences in the underlying scales, to consider the full basic physical equations directly in a numerical simulation. Then, reduced and simplified models have to be constructed, and their analytical properties and approximation qualities have to be investigated.
- The numerical simulation can study only specific configurations. For a qualitative understanding of the behaviour of the underlying system, the mathematical analysis of the underlying equations is indispensable.

In the mathematical analysis and the numerical appromixation of the partial differential equations describing nanostructures, several mathematical difficulties arise, e. g., the appropriate treatment of nonlinearities, simultaneous occurrence of continuous and discrete spectrum, multiple scales in space and time, and the ill-posedness of these problems.

Photonic crystals are materials which are composed of two or more different dielectrics or metals, and which exhibit a spatially periodic structure, typically at the length scale of hundred nanometers. Photonic crystals can be fabricated using processes such as photolithography or vertical deposition methods. They also occur in nature, e. g. in the microscopic structure of certain bird feathers, butterfly wings, or beetle shells.

A characteristic feature of photonic crystals is that they strongly affect the propagation of light waves at certain optical frequencies. This is due to the fact that the optical density inside a photonic crystal varies periodically at the length scale of about 400 to 800 nanometers, i. e., precisely at the scale of the wavelengths of optical light waves. Light waves that penetrate a photonic crystal are therefore subject to periodic, multiple diffraction, which leads to coherent wave interference inside the crystal. Depending on the frequency of the incident light wave this interference can either be constructive or destructive. In the latter case the light wave is not able to propagate inside the photonic crystal. Typically, this phenomenon only occurs for bounded ranges of optical wave frequencies, if it does occur at all. Such a range of inhibited wave frequencies is called a photonic band gap. Light waves with frequencies inside a photonic band gap are totally reflected by the photonic crystal. It is this effect which causes, e. g., the iridescent colours of peacock feathers.

In the mathematical modeling of photonic crystals by Maxwell's equations with periodic permittivity, such photonic band gaps are described as gaps in the spectrum of a selfadjoint operator with periodic coefficients, while the frequency ranges where constructive interference takes place form the spectrum (which is arranged in bands) of this selfadjoint operator.

In this proceedings volume we collect a series of lectures which introduce into the mathematical background needed for the modeling and simulation of light, in particular in periodic media, and for its applications in optical devices. We start with an introduction to Maxwell's equations, which build the basis for the mathematical description of all electro-magnetic phenomena, and thus in particular of optical waves. Next, we focus on explicit methods for the numerical computation of photonic band gaps. Furthermore, a general introduction to the so-called Floquet–Bloch theory is given, which provides analytical tools to investigate the spectrum of periodic differential operators and provides the aforementioned band-gap structure of selfadjoint operators with periodic coefficients, such as they occur for Maxwell's equations in a photonic crystal. In the rest of this volume we consider two applications. In the first application the theory of direct and inverse scattering is introduced and applied to periodic media, and the second application investigates nonlinear optical effects in wave guides which can be described by the nonlinear Schrödinger equation.

We greatly appreciate the opportunity to give this course in the framework of the Oberwolfach seminars, and we would like to thank the Oberwolfach institute for their kind hospitality. Further, we want to thank for the kind assistance of Birkhäuser in the realisation of these lecture notes.