

# 1

## Thermodynamic Cycles

### 1.1 Introduction to Thermodynamic Cycles

The concept of a thermodynamic cycle is based on depicting thermodynamic processes which involve the transfer heat and work. This is achieved by altering temperature, pressure, as well as other state variables; the cycle ultimately returning to its initial state. The fundamental basis of these cycles is the first law of thermodynamics which states that ‘energy cannot be created nor destroyed but only converted from one form to another’.

As shown in Figure 1.1, thermodynamic cycles are split into two primary classes – refrigeration cycles (also known as heat pump cycles) and power cycles such as the combustion engine cycle.

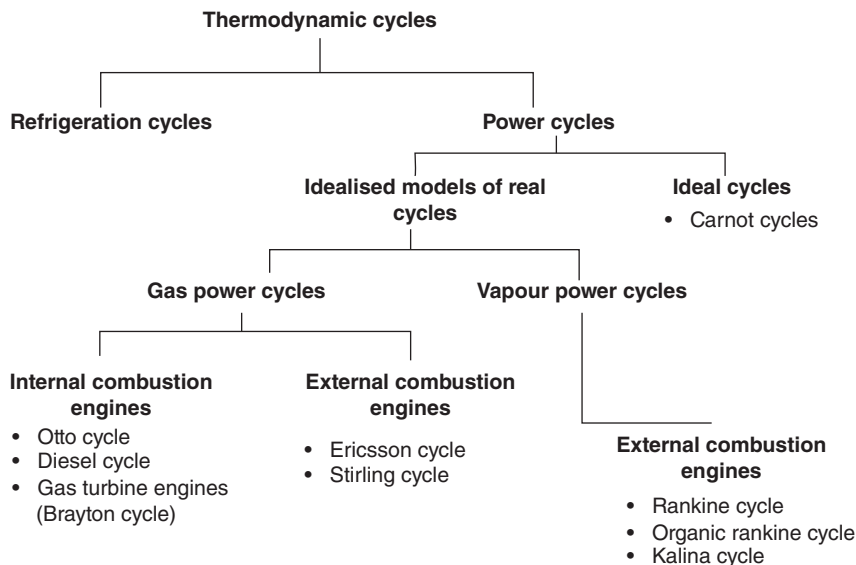
Cycles which transfer heat from low temperature to high temperature are classified as heat pump cycles, whereas cycles that convert heat input into mechanical work are designated as power cycles. And so, not inappropriately, thermodynamic power cycles provide the foundation for the operation of a heat engine. Power cycles are then further divided into groups depending on the type of heat engine. For cycles modelling internal combustion engines the groups are the Otto, Diesel, and Brayton cycles and for external combustion engines, they are the Rankine, Organic Rankine, and Kalina cycles.

### 1.2 Rankine Cycle

#### 1.2.1 Introduction

The Rankine cycle is a closed thermodynamic cycle that converts thermal energy into mechanical energy and is based on the fundamentals of the Carnot cycle proposed by the French physicist Nicolas Léonard Sadi Carnot in 1824. The Rankine cycle was proposed by the Scottish physicist William John Macquorn Rankine in 1865, and contrary to Carnot cycle – which is only a theoretical cycle – the Rankine cycle has many applications, several of which will be discussed further within this section.

The main application of the Rankine cycle is to convert thermal energy into electrical energy and this thermodynamic cycle is based on the rotation of a turbine



**Figure 1.1** Map of all thermodynamic cycles.

fixed to a shaft and on to which an electrical generator is also located. In this way the mechanical energy of the turbine is converted into electrical energy, a more versatile and widely used form of energy. Rankine cycles and their different configurations are extensively used in thermal power plants such as nuclear, gas, oil, coal, geothermal, biomass, and concentrated solar power plants [1].

This section aims to present and explore in depth the Rankine thermodynamic cycle and following a general description of the cycle and a reminder of the Carnot cycle, different configurations will be presented and illustrated by means of diagrams and schematics. The driver for these various enhanced configurations always being to improve either the energetic or the exergetic efficiency of the cycle (or both). In this respect energy is a quantitative parameter and exergy a qualitative parameter, and we need to measure both of these in order to determine the total useful work we can extract from a thermodynamic system. The equations used to model such systems will be highlighted and explained as these formulas help us to understand the different parameters that influence the overall performance of a system. Real-life examples will also be used to clarify these explanations, and relevantly, a section of this document also explores the link between Rankine cycles and thermal power plants.

### 1.2.2 Thermodynamic Diagrams

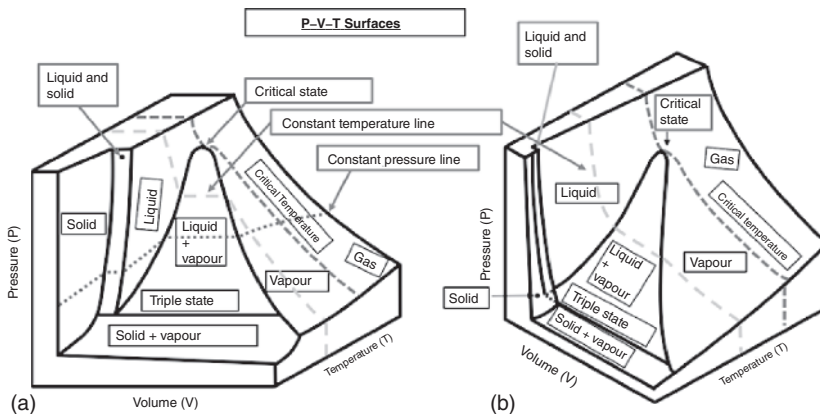
In order to clearly understand the following sections of this document, it is important to be familiar with the basics of thermodynamics – the first and second laws as well as thermodynamic diagrams. Indeed, these diagrams are key aids in helping to develop a deeper understanding about how thermodynamic cycles function. The three main

physical parameters used to describe a thermodynamic system are the temperature ( $^{\circ}\text{C}$  or  $\text{K}$ ), the pressure (bar or Pa), and volume ( $\text{m}^3$ ) and these give rise to three significant diagrams which can then be created to visualise the state of a fluid under specific conditions: the  $P$ - $V$  diagram, the  $T$ - $V$  diagram, and the  $P$ - $T$  diagram.  $P$ - $V$  diagrams plot pressure with respect to specific volume,  $T$ - $V$  diagrams plot temperature with respect to specific volume, and  $P$ - $T$  diagrams plot pressure with respect to temperature. Now as most thermodynamic cycles operate by making use of a phase change of the working fluid, such diagrams are also useful in that they graphically display values (of pressure, volume, and temperature) at which these phase changes occur, i.e. for a given material, the points when changes of state from say a solid to a liquid (the point of melting), or from a liquid to a gas (the point of vapourisation) occur – or their reverse processes – freezing and condensing. In fact, we can display the three parameters of pressure, volume, and temperature as a ‘surface’, Figure 1.2.

Firstly, you will note that we say ‘surface’ not ‘solid’, and that’s because at any particular temperature and pressure the condition of a material would only lie on the surface and not above nor below it. For example, take a point for say water in Figure 1.2, diagram (i), on the vapour section of the diagram. We might try to imagine a point above this – i.e. one with an increased pressure – but water vapour cannot exist at this higher pressure at that temperature and volume. Nor will it exist at a lower pressure. It can only exist at the values of pressure, temperature, and volume that are shown on the given surface [2].

So, what are the three single-phase regions we can see on the diagrams? They are the gas/vapour region, the liquid region, and the solid region, and between these we have the transition or two-state phase change regions where the solids are melting or freezing, and the liquids are evaporating or condensing.

Now when considering a vapour and a gas, because the two terms are often confused – and to some extent are synonymous in that they both refer to a physical state that is not solid or liquid – we perhaps explain the need to differentiate them. Technically, below the critical temperature, a vapour and its liquid are in



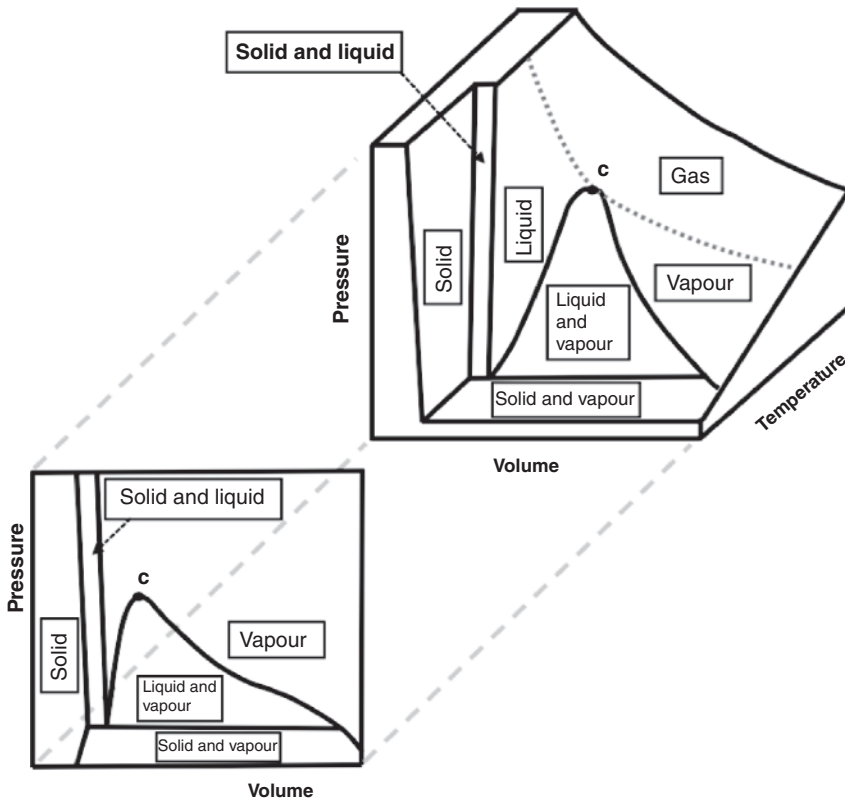
**Figure 1.2**  $P$ - $V$ - $T$  diagrams. (a) A material that contracts on freezing. (b) A material that expands on freezing – water being the most well-known example.

'equilibrium' and a vapour can condense back to a liquid with a small change of pressure or temperature. Whereas a gas cannot condense to a liquid without changes in both pressure and temperature, and above the critical temperature line the liquid state does not exist at all! But what do we mean by being 'in equilibrium'?

Well, evaporation from the surface of a liquid or fluid takes place when the liquid molecules have enough momentum to overcome the cohesive forces within the fluid and escape into the space above it. To all intents and purposes, it becomes a gas, but because both the liquid and gas are present together it is more accurately known (in this state) as a vapour. If we now add heat to the liquid the momentum of the molecules increases, and more liquid evaporates. Similarly, a reduction in pressure above the liquid also reduces the momentum necessary for molecules to escape and so again evaporation is increased. But if the container which holds the liquid is sealed so that the evaporated vapour cannot escape, then the amount of liquid that can evaporate is finite. That is at any fixed value of temperature and pressure the amount of vapour will rise to a steady fixed amount and then no more can be added and the vapour is said to be 'fully saturated'. If any more molecules reach sufficient momentum to escape the surface then their number is exactly matched by those condensing back into the liquid, so an equilibrium or balance occurs between the two processes to limit the amount of vapour – or more correctly – the vapour pressure. Raise the temperature and the amount of vapour that can be maintained increases, and the 'fully saturated' line – the dome or bell-shaped line (in the earlier diagrams) illustrates the limit of this. So, to the right-hand side of the curve is where all the liquid will have evaporated and the liquid state cannot exist, whilst the left-hand side shows where all the solid has melted to liquid and the solid state can now longer exist. But under the curve is the region where both liquid and vapour exist together. Until that is, we reach the peak or 'critical point' or critical state where the distinction between the liquid and the gas no longer exists. Here the molecules are evaporating so rapidly that the density of the liquid and the vapour are equal and produce a 'supercritical fluid'; the point being that condensation of the gas will never occur above the critical point no matter how much pressure we apply.

But what else does the  $P$ - $V$ - $T$  diagram show us? For example, if we look at the liquid and solid regions, we will notice just how steep these surfaces are, and that's because, compared to gases, liquids and certainly solids are much harder to compress. So, at a particular temperature, if we increase the pressure it does not really change the volume or, conversely, the pressure will rise dramatically for a small change in volume. We can also note the transition phase between a solid and a vapour where 'sublimation' can occur. Dry ice is an example of such a material and is used where it is necessary to keep items cold but where any packaging will not get wet with liquid as the 'dry' ice warms up. We can also notice the triple state line where all three phases can coexist.

However, as useful as the  $P$ - $V$ - $T$  surface plot is to indicate the various states and transition phases of a substance they are a bit difficult to read-off from on to value scales. So, it is more usual to present thermodynamic information by means of two-axis diagrams that we briefly noted earlier. For example, the  $P$ - $V$  diagram is the



**Figure 1.3**  $P$ - $V$  perspective of a  $P$ - $V$ - $T$  diagram.

view we would obtain by looking at the surface from a pressure–volume perspective to see how a thermodynamic process changes with pressure and volume, Figure 1.3.

Usually we are only interested in the saturated liquid and saturated vapour areas, so our  $P$ - $V$  diagrams are often ‘truncated’. But in addition, our volume axis is more usually scaled in terms of *specific volume*, i.e. the cubic volume per kilogram of mass rather than just volume, because then, as an intensive property, the value does not depend on the size of the thermodynamic system. Hence, as we can see in Figure 1.4, the  $P$ - $V$  diagram (also called a Clapeyron diagram) presents pressure as a function of the specific volume ( $\text{m}^3/\text{kg}$ ), and on this diagram the left-hand side of the vapour dome shows the limit between the liquid region whilst the right-hand side shows the limit between the liquid/vapour region and the superheated vapour region. The diagram also denotes the plot of two isothermal lines or lines of constant temperature demonstrating that, inside the dome, the phase transition between liquid and vapour is a constant-pressure, constant-temperature process.

The other two diagrams that we can better visualise as two-dimensional graphs are the temperature–volume and the pressure–temperature diagrams as shown in Figure 1.5.

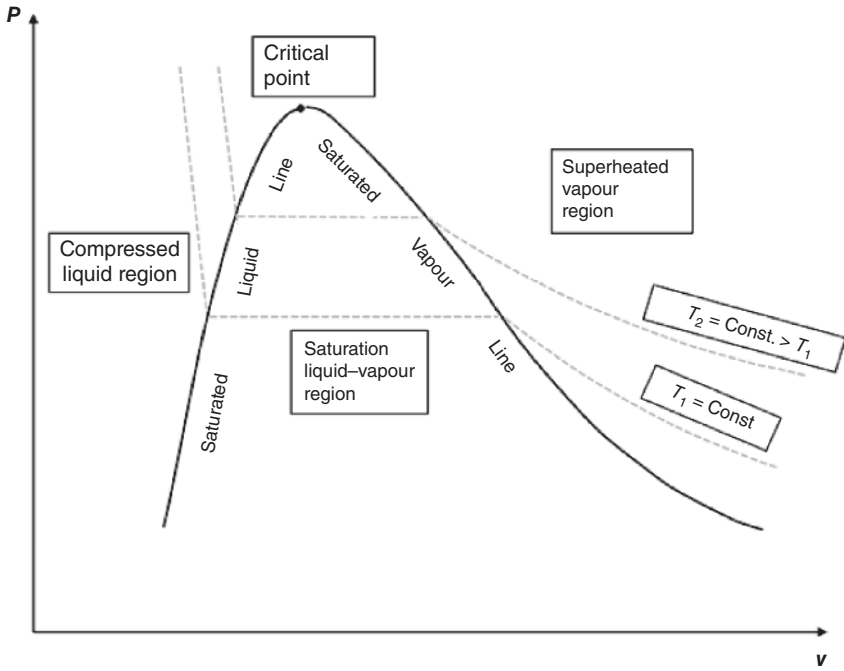


Figure 1.4  $P$ - $V$  diagram of a pure substance.

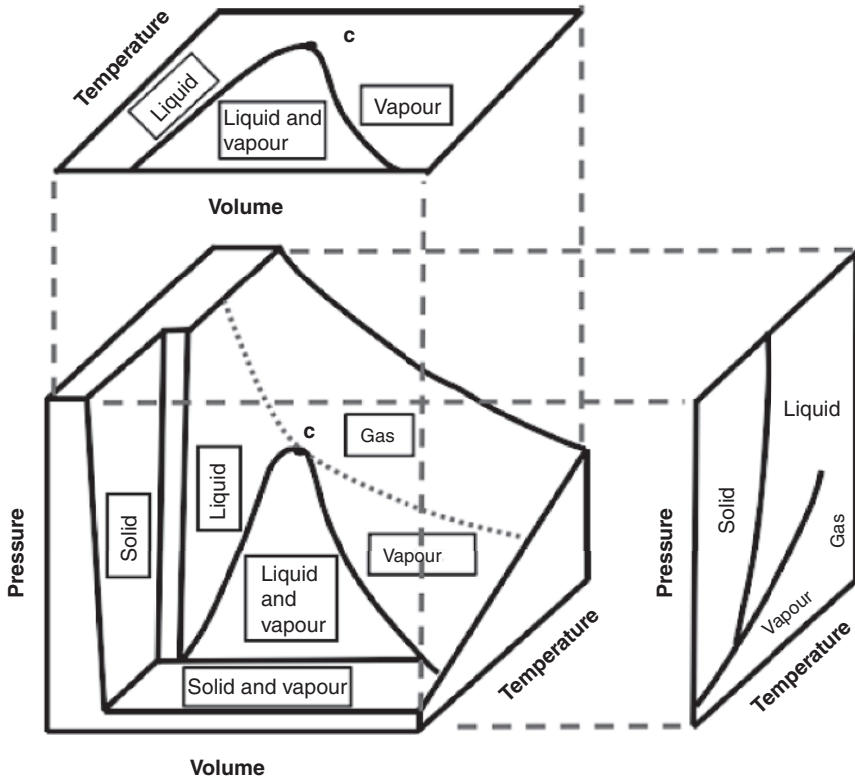
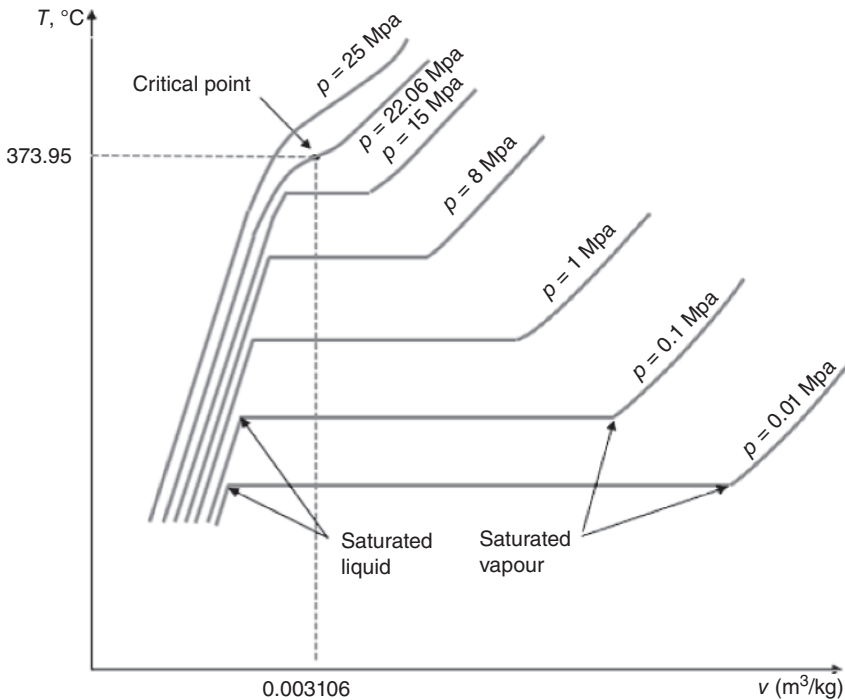


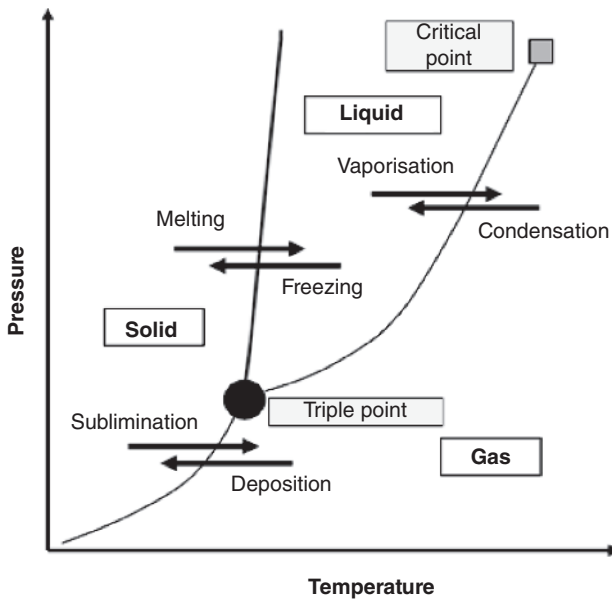
Figure 1.5  $T$ - $V$  and  $P$ - $T$  perspectives of a  $P$ - $V$ - $T$  diagram.



**Figure 1.6**  $T$ - $V$  diagram of a pure substance.

The vapour dome can also be seen on the  $T$ - $V$  diagram in Figure 1.6, however, it is not directly visible on the  $P$ - $T$  diagram in Figure 1.7 – which is also known as a phase diagram – in that for a given pressure and temperature the state of the substance is fully defined. As phase transitions occur under constant pressure and temperature for pure substances, alteration of the pressure or temperature will result in a phase change for the substance, the outcome being either a pure liquid or a pure vapour. For that reason, the liquid–vapour region is just a line on the  $P$ - $T$  diagram and the vapour dome is not visible because, as we have already seen, a third dimension (the  $P$ - $V$ - $T$  diagram) is required to fully visualise the state of the fluid.

Two other thermodynamic variables are also very useful in describing a thermodynamic system from energetic and exergetic viewpoints, and these are the specific enthalpy  $h$  (kJ/kg) and the specific entropy  $s$  (kJ/kg K). Enthalpy ( $h$ ) measures the total heat content of a system and is equal to the internal energy (i.e. the energy related to the molecular structure or state of the material and the degree of molecular activity) which is then added to the product of pressure and volume. In the analysis of systems that involve fluid flow, we often meet the combination of these two properties, defined from  $h = u + PV$ , where  $u$  represents the macroscopic energy of the non-flowing fluid and  $PV$  represents the energy needed to push and maintain the fluid flow around the system. But the second term, entropy, is defined as a measure of randomness of molecules, so what, we may ask, is its relevance to thermodynamics?



**Figure 1.7**  $P$ - $T$  diagram of a pure substance.

In fact, entropy is very relevant to thermodynamics and is the function of the quantity of heat that can be converted into work – and yes, it still does indicate the randomness of molecules. For example, suppose we put 100J of energy into a system trying to convert this all into work can we then get 100J back out? The answer of course is No! Some of that energy will be used in making more molecules move at random or increasing the momentum of those already in motion – that’s why the entropy of a solid is less than that of a gas because gases have more molecules moving at random. Temperature increases the entropy of a material because as the temperature rises, the molecules spread out and take up more room – that is, they become more disordered. Pressure, on the other hand, has the opposite effect, it forces molecules closer together, so they become more ordered and entropy decreases. However, we should note that solids and liquids are nearly incompressible, so any entropy decrease is minimal.

So, in addition to those already mentioned, many other two- and three-dimensional diagrams can be plotted. For example, the  $T$ - $s$  diagram (temperature as a function of specific entropy) and the  $P$ - $h$  diagram (pressure as a function of specific enthalpy) are both very useful diagrams. An example of a temperature-specific entropy ( $T$ - $s$ ) diagram can be seen in Figure 1.8, and is often used to describe thermodynamic power cycles since the area drawn by the cycle on this form of diagram is equal to the work provided by it. Alternatively, a pressure–enthalpy ( $P$ - $h$ ) diagram, also called a Mollier diagram, might be used, as shown in Figure 1.9, to help describe receptor cycles such as refrigeration cycles.

These diagrams are also essential tools to aid understanding and for describing thermodynamic cycles, but in most cases when working with Rankine cycles the

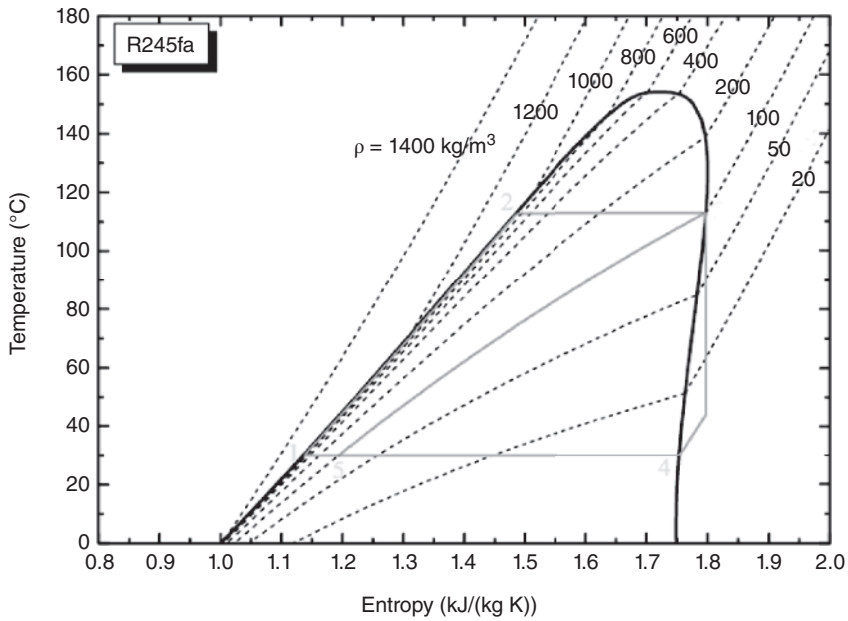


Figure 1.8 Example  $T-s$  diagram. Source: Based on Janie et al. [3].

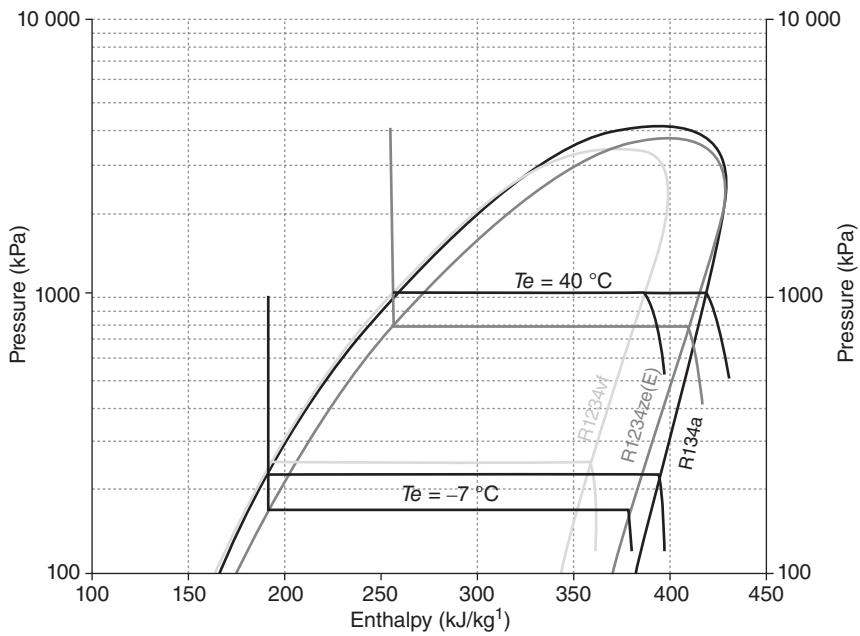


Figure 1.9  $P-h$  diagram of refrigerants. Source: Sethi et al. [4].

$T$ - $s$  diagram is the most suitable. It will therefore be used extensively in the sections that follow.

### 1.2.3 The Carnot Cycle

Before we discuss the Rankine cycle and its application, we first need to consider its inspiration – the Carnot cycle [5].

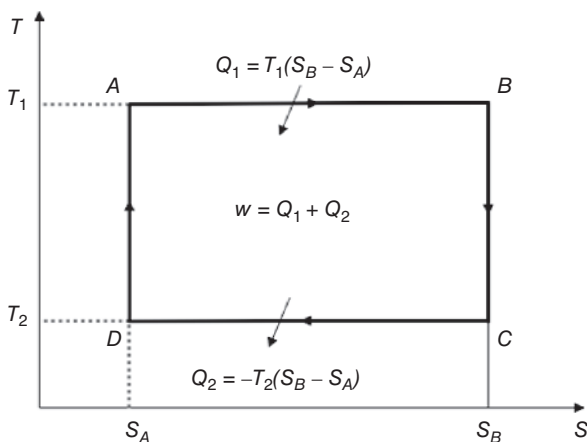
Nicholas Léonard Sadi Carnot, the French mechanical engineer and physicist, is often described as the ‘father of thermodynamics’. He published only one book, but in it he expressed the first successful theory of maximum heat engine efficiency and so laid the foundations, for what was then, an entirely new discipline. The Carnot cycle is therefore used as the common reference for every thermodynamic power cycle because even though the Carnot cycle is only a theoretical model, it does provide the standard as the most efficient engine possible. In other words, because the thermal efficiency of the Carnot cycle only depends on the absolute temperatures of the two heat reservoirs that drive the process, it gives us the upper limit of both energetic and exergetic thermodynamic cycle efficiency [6].

Most commonly, the working fluid used to explain the Carnot cycle is water and the cycle is divided into four phases as shown in Figure 1.10:

1. Isothermal expansion, (AB)
2. Isentropic (constant entropy) expansion, (BC)
3. Isothermal (constant temperature) compression, (CD)
4. Isentropic (constant entropy) compression, (DA)

Theoretically, these four processes are reversible, but in reality, reversible transformations do not exist because of, e.g. heat losses due to friction. The figure indicates a clockwise cycle and so is known as a Carnot ‘power’ cycle (power cycles are those which convert heat into mechanical work output whilst a ‘heat pump’ cycle transfers heat from lower to higher temperatures using mechanical work input).

Thermodynamic power cycles are the operating basis of heat engines, which currently supply much of the world’s electric power and run almost all motor vehicles.



**Figure 1.10**  $T$ - $s$  diagram of the Carnot cycle.

Power cycles are classified according to the type of heat engine they model, e.g. we have the Otto cycle and the Diesel cycle for modelling internal combustion engines, the Brayton cycle for modelling gas turbines, and the Rankine cycle for modelling steam turbines.

Consider for a moment, the above  $T$ - $s$  diagram as representing the Carnot cycle for an ideal or perfect gas turbine, i.e. one with no friction and the necessary heat transfers occurring without loss. What do the four stages of our cycle represent?

Essentially, we start with two 'heat reservoirs' with temperatures  $T_1$  and  $T_2$ , respectively, and these have such a large thermal capacity that their temperatures are stable and unaffected by the cycle, which takes place as follows:

- Stage 1 (AB): Heat is transferred from the high-temperature reservoir ( $T_1$ ) at constant temperature. During this phase, the entropy of the gas is increased by allowing it to expand and reduce its pressure and in doing so producing work on the surroundings – e.g. moving a piston or rotating a shaft within a turbine.
- Stage 2 (BC): During this stage of the cycle, it is assumed that the gas is thermally insulated from both the hot and cold reservoirs, so that they neither gain nor lose heat from the gas – i.e. the conditions are adiabatic (Greek  $\equiv$  without loss). Instead, the gas continues to expand (and do more work) further reducing the pressure and losing internal energy in an amount equal to the work done. Since this gas expansion occurs without any heat input, it cools to the 'cold' temperature  $T_2$  but with the entropy unchanged.
- Stage 3 (CD): At this point the gas is now in thermal contact with the cold reservoir at temperature  $T_2$ , and work is then done on the gas to compress it – e.g. the piston is forced down or the gas passes into a compressor. In doing so, heat energy is transferred to the cold reservoir and the entropy of the system decreases by the same amount that it gained at Stage 1.
- Stage 4 (DA): It is assumed that the gas is thermally isolated from the hot and cold reservoirs (and any mechanical processes are frictionless) and further compression takes place, increasing the internal energy and so causing the temperature to rise back to  $T_1$ . This assumes that this temperature rise is solely due to the work added to the system and so the entropy remains unchanged. The gas is now back at the same state as the start of Stage 1.

As such, the work (kJ) provided by the thermodynamic cycle is given by:

$$W = \oint PdV = \oint Tds \quad (1.1)$$

In other words, the total work is given by integrating and thereby accumulating all the infinitesimally small changes in volume and pressure or entropy and temperature the gas makes on its journey around the cycle. Of course, the path taken by a real process differs somewhat from the ideal processes shown, but the idea here is to produce a perfect standard against which all other heat engines can be judged.

In a  $T$ - $s$  diagram, the area under the upper portion of the figure represents the thermal energy absorbed during the cycle, whilst the area under the lower portion represents the thermal energy removed during the cycle – hence the area *inside*

the cycle will represent the difference between these two. However, since the internal energy of the system must return to its initial value, this difference must also represent the total amount of work performed by the system over one cycle. So, in definitive terms on the diagram, the absolute value of *all* this work ( $|W|$ ) corresponds to the area of the rectangle. Or:

$$|W| = (T_1 - T_2) \cdot (S_B - S_A) \quad (1.2)$$

If  $Q_H$  represents the total amount of thermal energy transferred from the hot reservoir to the system (kJ), and  $Q_C$  represents the total amount of thermal energy transferred from the system to the cold reservoir (kJ), both as given by Eqs. (1.3)–(1.5):

$$Q_H = T_1 \cdot (S_B - S_A) \quad \text{and} \quad Q_C = T_2 \cdot (S_B - S_A) \quad (1.3)$$

Then the efficiency of the Carnot cycle can be defined as:

$$\eta = \frac{|W|}{Q_H} = \frac{Q_H - Q_C}{Q_H} = \frac{(T_1 - T_2) \cdot (S_B - S_A)}{T_1 \cdot (S_B - S_A)} \quad (1.4)$$

where,  $T_1$  is the hot source temperature and  $T_2$  is the cold source temperature (both are expressed in [K] Kelvin), and  $S_B$  and  $S_A$  correspond, respectively, to the entropy (kJ/K) at points B and A. From which, the overall efficiency will be given by the following equation:

$$\eta = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad (1.5)$$

### 1.2.4 Ideal and Actual Rankine Cycles

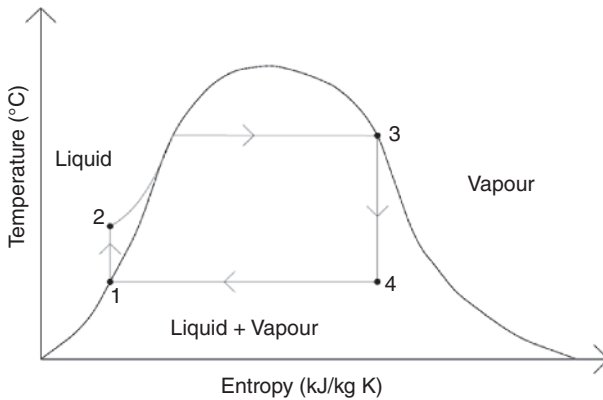
The Carnot cycle is just a theoretical cycle. It is not possible to build a real engine working under such conditions because of real component *irreversibility*. For example, the transformations result in heat dissipation through friction. So, these transformations are defined as irreversible because once they have occurred, it is impossible to return them to their initial state. A reversible transformation is a succession of infinitesimal changes, each occurring *without* any energy loss, and in practice, this kind of transformation does not exist in the real world even though each component is designed in order to approach an irreversible behaviour.

The Rankine cycle is very similar to the Carnot cycle, the difference being that the reversible isothermal transformations are now replaced by isobaric (constant pressure) ones. In most cases, the working fluid is water too.

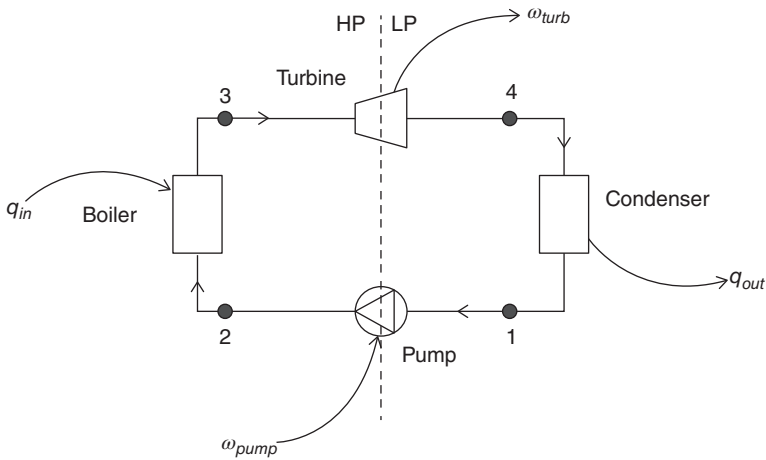
The ideal Rankine cycle is composed of two isobaric transformations and two adiabatic and reversible isentropic (constant entropy) transformations as shown in Figure 1.11. The four main components required for the Rankine cycle are a pump, a boiler, a turbine, and a condenser as shown in Figure 1.12.

The four transformations involved in the cycle are:

- 1–2: Isentropic compression (pump). The fluid (in liquid state) is compressed while the specific entropy remains constant.



**Figure 1.11**  $T$ - $s$  diagram of the Rankine cycle.



**Figure 1.12** Schematic of the Rankine cycle.

- 2–3: Isobaric heat addition inducing vapourisation of the fluid (boiler), i.e. the fluid is heated under constant pressure and this results in a phase transition – the fluid changing from liquid to a mixture of liquid and vapour.
- 3–4: Isentropic expansion (turbine). The fluid is expanded through a turbine while the specific entropy again remains constant.
- 4–1: Isobaric heat rejection inducing liquefaction of the fluid (condenser), i.e. the fluid is cooled down under constant pressure and the vapour condenses.

#### 1.2.4.1 Ideal Cycle

To describe the Rankine thermodynamic cycle, five physical variables are required:

1. The pressure  $P$  (Pa)
2. The temperature  $T$  (K)

3. The steam quality  $x$ , which represents the proportion of saturated vapour in a liquid/vapour mixture.

$$x = \frac{m_{\text{vapour}}}{m_{\text{vapour}} + m_{\text{liquid}}} \quad (1.6)$$

where,  $m_{\text{vapour}}$  and  $m_{\text{liquid}}$  correspond, respectively, to the masses (kg) of the vapour and liquid contained in the mixture. (For example, at point number 1 on the  $T$ - $s$  diagram,  $x(1) = 0$ , i.e. the fluid is a fully saturated liquid.)

4. The specific enthalpy of the fluid  $h$  (kJ/kg)  
5. The specific entropy of the fluid  $s$  (kJ/kg K)

Knowing the values of each of these variables at every point of the diagram is very useful, and helpfully the five variables are linked to each other through physical relationships.

Looking at the diagram it may not be obvious, but the cycle is divided in two pressure areas: high pressure (HP) and low pressure (LP) – these pressures being expressed in Pascal (Pa). Points 2 and 3 are in the HP section and points 4 and 1 are in the LP section, and as the transformation between 2 and 3 is isobaric:  $P_2 = P_3$ . (And similarly, for the same reason,  $P_1 = P_4$ .) Then the work provided to the system corresponds to the area of the quadrilateral shape drawn by the four points, but to calculate this work, the specific enthalpy of the fluid must be used. From which we can then determine the *specific work* ( $\omega$ ) – which corresponds to the amount of work delivered per unit of mass (kJ/kg).

The thermodynamic system is exchanging energy (work or heat) with its environment, and we should note here the convention that: heat transferred *to* a fluid is a positive quantity; heat transferred *from* a fluid is a negative quantity. It is necessary to note this because a Rankine cycle uses part of its work to compress the water, so will need to deduct the power put into the pump from the power output of the turbine in order to obtain the net work. Accordingly, the required work and heat parameters are determined from:

$\omega_{\text{turb}}$ , the work given by the fluid to the turbine (kJ/kg). It is negative as the fluid is giving work:

$$\omega_{\text{turb}} = h_4 - h_3 \quad (1.7)$$

$\omega_{\text{pump}}$ , the work given by the pump to the fluid (kJ/kg). It is positive as the pump gives work to the fluid:

$$\omega_{\text{pump}} = h_2 - h_1 \quad (1.8)$$

$q_{\text{in}}$ , the heat given by the boiler to the fluid (kJ/kg). It is positive as the boiler gives heat to the fluid:

$$q_{\text{in}} = h_3 - h_2 \quad (1.9)$$

$q_{\text{out}}$ , the heat given by the fluid to the condenser (kJ/kg). It is negative as the fluid rejects heat:

$$q_{\text{out}} = h_1 - h_4 \quad (1.10)$$

So, as the net work ( $\omega_{net}$ ) (kJ/kg) provided by the cycle is governed by:

$$\omega_{net} = \omega_{turb} + \omega_{pump} \quad (1.11)$$

Then the thermal efficiency ( $\eta_{th}$ ) of the ideal Rankine cycle can be defined by:

$$\eta_{th} = \frac{|\omega_{turb}|}{q_{in}} \quad (1.12)$$

And hence the overall efficiency ( $\eta$ ) of the ideal Rankine cycle can be defined by:

$$\eta = \frac{|\omega_{net}|}{q_{in}} = \frac{|\omega_{turb}| - \omega_{pump}}{q_{in}} \quad (1.13)$$

Substituting Eqs. (1.4)–(1.6) then leads to:

$$\eta = \frac{|h_4 - h_3| - (h_2 - h_1)}{h_3 - h_2} = \frac{h_3 - h_4 - (h_2 - h_1)}{h_3 - h_2} \quad (1.14)$$

In practice however, compared to the output of the turbine, the work input to the pump is very small. In which case, if  $\omega_{pump}$  can be neglected compared to the other coefficients, we arrive at the useful approximation of:

$$\eta \approx \frac{h_3 - h_4}{h_3 - h_2} \quad (1.15)$$

So to estimate the efficiency of the system, we can also use the first law of thermodynamics which states that the net energy of a system is equal to 0, i.e.

$$\omega_{turb} + \omega_{pump} + q_{in} + q_{out} = 0 \quad (1.16)$$

And further assuming  $\omega_{pump}$  is neglectable compared to the other coefficients gives:

$$|\omega_{turb}| \approx q_{in} - |q_{out}| \quad (1.17)$$

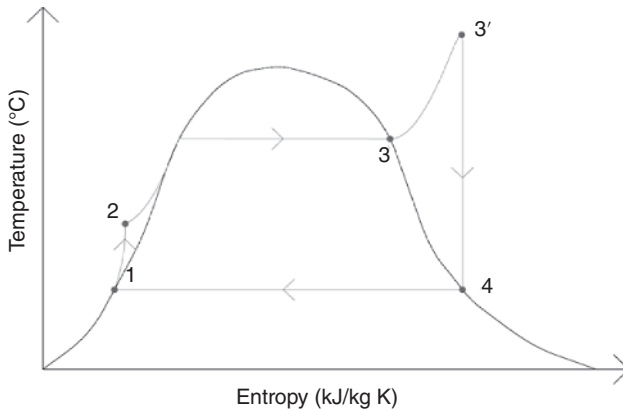
From which the efficiency of the ideal Rankine cycle can now be estimated from the following formula:

$$\eta \approx \frac{|\omega_{turb}|}{q_{in}} \approx \frac{q_{in} - |q_{out}|}{q_{in}} = 1 - \frac{|q_{out}|}{q_{in}} \quad (1.18)$$

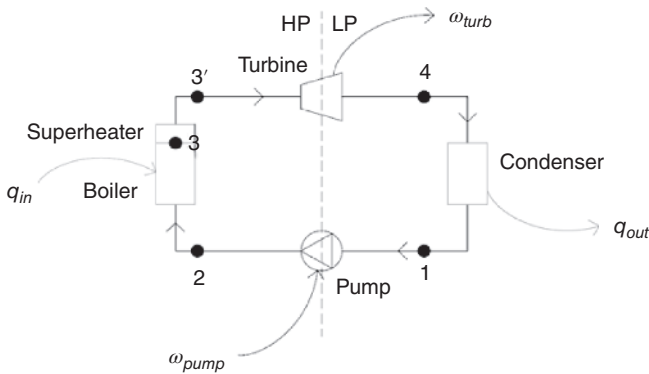
Practically the efficiency of the ideal Rankine cycle is necessarily lower than the efficiency of the theoretical Carnot cycle even using the same hot and cold sources. In addition, the cycle presented in this section is still an ideal cycle, and the actual cycle used, for instance, for power plant analysis is slightly different.

#### 1.2.4.2 Superheated Rankine Cycle

In the conventional Rankine cycle, the isentropic expansion occurs in the wet steam domain (transformation 3–4 in Figure 1.11). To explain, when a boiler heats water, bubbles breaking through the water surface pull tiny droplets of water into the steam which then becomes partially wet ('wet steam') from the added liquid. As water is the working fluid, it can cause damage to the turbine due to corrosion. Therefore, to mitigate this in power plants, the vapour is superheated to remove any moisture that it contains. Steam dryness is important because it has a direct effect on the total



**Figure 1.13**  $T$ - $s$  diagram of the superheated Rankine cycle.



**Figure 1.14** Schematic of the superheated Rankine cycle.

amount of transferable energy contained within the steam. For example, saturated steam (100% dry) contains 100% of the latent heat available at that pressure.

The result of this can be seen in the shape variation for the  $T$ - $s$  diagram of the cycle as shown in Figure 1.13. In this case, the cycle is still a Rankine cycle and is sometimes called Hirn (or Rankine with superheating) cycle, so one extra component is required: the superheater as shown in Figure 1.14. However, in most cases, the boiler and the superheater are the same component.

Overheating is also interesting because it increases the amount of work given to the turbine. As we can see on the  $T$ - $s$  diagram, the area of the superheated cycle is bigger than the area of the classical Rankine cycle. And in this way, for a same power output, the water flow rate can be lower for the Rankine-Hirn cycle than for a classical Rankine cycle, so improving the energetic efficiency of the system. Unfortunately, superheating also increases the irreversibility of the cycle and therefore, correspondingly, the exergetic efficiency of the system is regrettably reduced.

The main difference between the two cycles is that the heat given to the fluid by the boiler/superheater enhances the superheated cycle. On the  $T$ - $s$  diagram, this

extra heat addition corresponds to the transformation 3–3'. This way, the expansion of the fluid then occurs within a superheated steam domain and corresponds to the transformation 3'–4 on the diagram. Furthermore, this transformation is assumed to display a greater degree of isentropic change and so is shown as a vertical line on the diagram, contrary to the transformation we now see between 1 and 2.

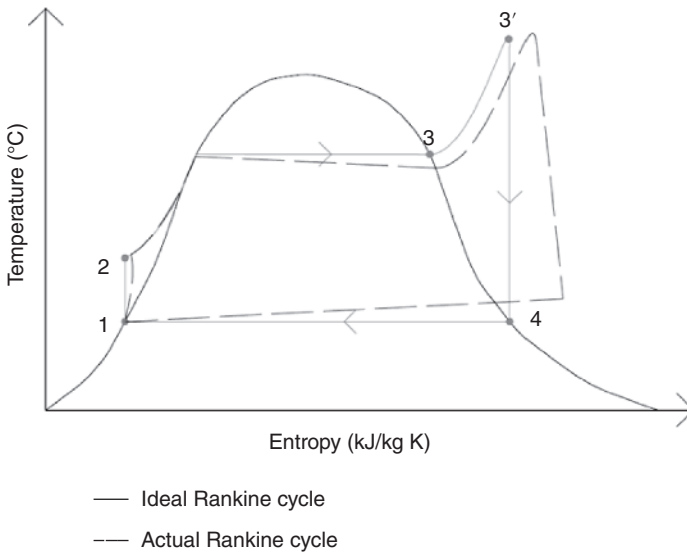
The efficiency of the superheated cycle is given by the same equation as the classical Rankine cycle (Eqs. (1.13) and (1.14)) – the difference being in the position of point 3' (instead of 3), i.e.

$$\eta = \frac{h'_3 - h_4 - (h_2 - h_1)}{h'_3 - h_2} \quad (1.19)$$

### 1.2.4.3 Actual Rankine Cycle

The *actual* Rankine cycle is the best cycle with which to model the transformations occurring within a power plant as it is the one which takes both system irreversibility and system imperfections into consideration.

In this cycle, friction caused by the moving fluid induces pressure losses or heat loss, and as a result the isobaric transformations described in the ideal Rankine cycle are not exactly isobaric in the actual cycle. Pressure drops in the condenser and in the turbine reduce the area of the cycle, as shown on the  $T$ – $s$  diagram in Figure 1.15. Hence, the specific work of the cycle is lower for the actual cycle than for the ideal cycle. Furthermore, the isentropic transformations described in the ideal cycle are not reversible, as they are in the actual cycle: irreversibility deteriorates the performances of the pump and of the turbine. Hence, the exergetic efficiency is reduced.



**Figure 1.15** Ideal Rankine cycle vs. actual Rankine cycle on a  $T$ – $s$  diagram.