And so:

$$E(t) = \frac{(t-2)^2}{2.6667}$$

Following the definition of $F(t)$:

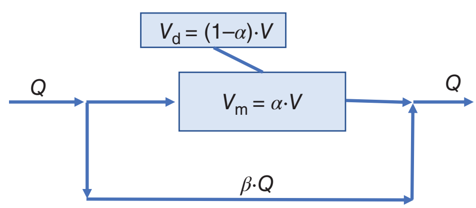
$$F(t) = \int_0^t E(t)\mathrm{d}t = \int_0^t \frac{(t-2)^2}{2.6667}\mathrm{d}t = 0.124\,984t^3 - 0.749\,906t^2 + 1.499\,81t + K$$

being "K" an integration constant. By definition, we know that:

$$\lim_{t\to\infty} F(t) = 1 = \lim_{t\to 2} F(t)$$

so the constant "K" should be equal to zero.

Problem 1.18 A reaction system is described by the following model:



At a given moment, the system presents values of $\alpha = 0.1$ and $\beta = 0.9$. A series of reforms are carried out in the system, and it is achieved that $\alpha = 0.9$ and $\beta = 0.1$.

(a) Qualitatively draw the response of the system to a tracer impulse, comparing the two situations.
(b) Also, draw the corresponding $F(t)$ curves for both situations.
(c) Indicate what reforms had to be made to achieve this change.

Solution to Problem 1.18

(a) In this system, the residence time compared to that expected is:

$$\overline{t_{\text{actual}}} = \frac{V_m}{Q_{\text{reactor}}} = \frac{\alpha V}{(1-\beta)Q} = \frac{\alpha}{(1-\beta)}\overline{t_{\text{expected}}}$$

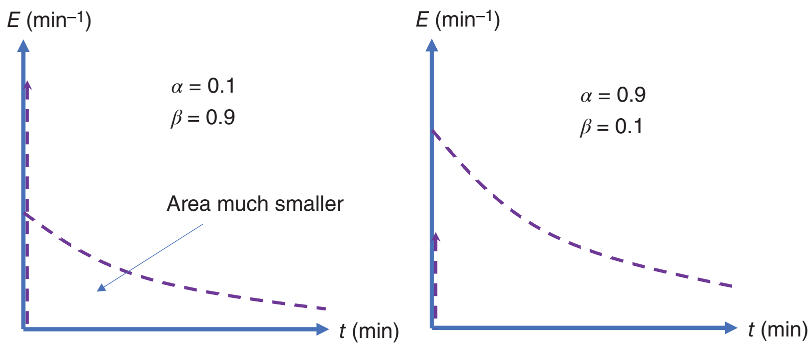
As we have that $\alpha = 0.1$ and $\beta = 0.9$ at the beginning:

$$\overline{t_{\text{actual}}} = \frac{0.1}{(1-0.9)}\overline{t_{\text{expected}}} = \overline{t_{\text{expected}}}$$

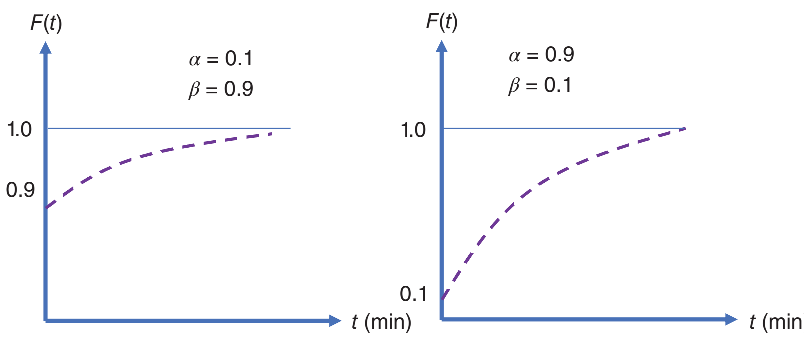
In the second case:

$$\overline{t_{\text{actual}}} = \frac{0.9}{(1-0.1)}\overline{t_{\text{expected}}} = \overline{t_{\text{expected}}}$$

So it will be very difficult to observe these changes. Nevertheless, the bypass produces a very important peak at time equal to zero in both situations, but the amount of tracer passing through the bypass is much higher in the first case. We can have something similar to:



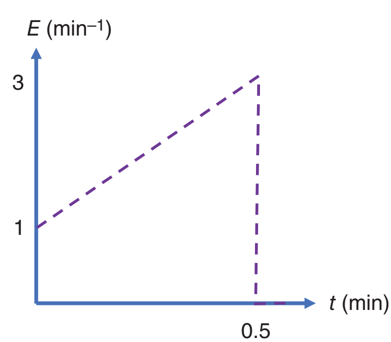
(b) The cumulative time function would have the following aspect:



(c) Usually, a better agitation is procured to eliminate dead volumes. Also, reduction of bypass flow is achieved with better agitation or by changing the reactor filling.

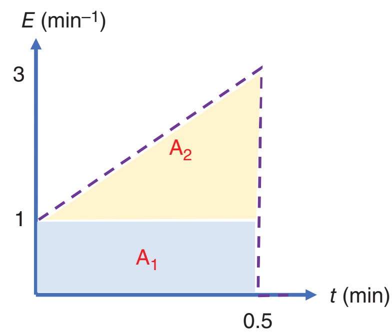
Problem 1.19 In a tracer input experiment, a perfect pulse is injected, and the following distribution is obtained:

(a) Calculate the mean residence time of the reactor.
(b) What will the output signal be like if the tracer has been injected in step?



Solution to Problem 1.19

(a) In the signal obtained, the total area is $1{\cdot}0.5 + (0.5{\cdot}2/2) = 1$ (just checking).

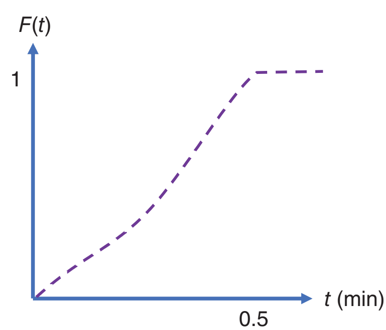


The straight-line connecting points (0, 1) and (0.5, 3) is $E(t) = 1 + 4t$, so:

$$t_m = \int_0^{\infty} t \cdot E(t)\mathrm{d}t = \int_0^{0.5} t \cdot (1 + 4t)\mathrm{d}t = 0.2917\ \mathrm{min}$$

(b) If a step tracer run is done, we will get:

$$F(t) = \int_0^{t} E(t)\mathrm{d}t = \int_0^{t} (1 + 4t)\mathrm{d}t = 2t^2 + t$$



Problem 1.20 A commercial-scale distillation tray is being studied for residence time using a fiber optic technique. A pulse of a 10 g/l solution of Rhodamine-B dye was injected into the downcomer of the top tray. The response data, summarizing the time (min) and corresponding output voltage (V), are provided as follows:

Time (min)	Output voltage (V)
0.0	0.00
0.1	0.00
0.2	2.80
0.3	4.48

Time (min)	Output voltage (V)
0.4	3.32
0.5	1.70
0.6	0.84
0.7	0.39
0.8	0.18
0.9	0.11
1.0	0.07
1.1	0.03
1.2	0.04
1.3	0.01
1.4	0.01
1.5	0.01
1.6	0.00
1.7	0.01
1.8	0.00
1.9	0.00

Note: The output voltage is proportional to the concentration of the dye.

(a) Determine the mean residence time and the characteristic $F(t)$ curve for this system.
(b) Use the dispersion model and the variance method to determine Bo (Bodenstein number) and Pe (Peclet number).
(c) Estimate the number of equal-sized CSTRs in series that would exhibit a comparable dispersion to the observed experimental data.
(d) Provide comments on the obtained results.

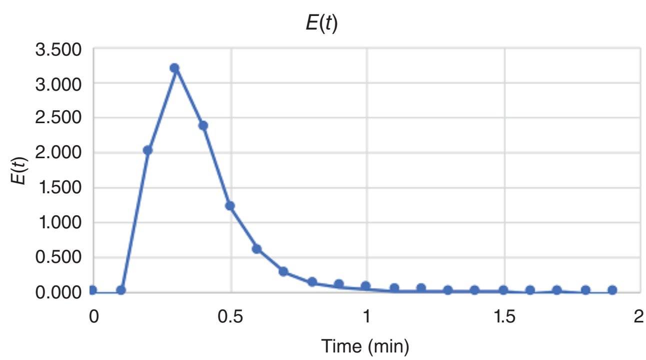
Solution to Problem 1.20

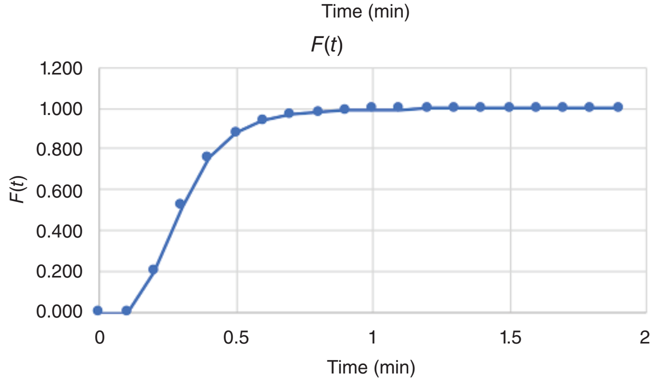
For details refer the Wiley website at http://www.wiley-vch.de/ISBN9783527354115

(a) Using similar techniques to those mentioned in other problems, we easily find:

$C(t)$	$C(t)\cdot\Delta t$	$E(t)$	$E(t)\cdot\Delta t$	$F(t)$	$t\cdot E(t)$	$t\cdot E(t)\cdot\Delta t$	$(t-t_m)^2\cdot E(t)$	$(t-t_m)^2\cdot E(t)\cdot\Delta t$
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.140	0.000	0.100	0.000	0.000	0.000	0.000	0.000
2.800	0.364	2.000	0.260	0.200	0.400	0.020	0.064	0.003
4.480	0.390	3.200	0.279	0.520	0.960	0.068	0.020	0.004
3.320	0.251	2.371	0.179	0.757	0.949	0.095	0.001	0.001
1.700	0.127	1.214	0.091	0.879	0.607	0.078	0.018	0.001

C(t)	**C(t)·Δt**	**E(t)**	**E(t)·Δt**	**F(t)**	**t·E(t)**	**t·E(t)·Δt**	**$(t-t_m)^2$· E(t)**	**$(t-t_m)^2$· E(t)·Δt**
0.840	0.062	0.600	0.044	0.939	0.360	0.048	0.029	0.002
0.390	0.029	0.279	0.020	0.966	0.195	0.028	0.029	0.003
0.180	0.015	0.129	0.010	0.979	0.103	0.015	0.023	0.003
0.110	0.009	0.079	0.006	0.987	0.071	0.009	0.021	0.002
0.070	0.005	0.050	0.004	0.992	0.050	0.006	0.019	0.002
0.030	0.004	0.021	0.003	0.994	0.024	0.004	0.011	0.002
0.040	0.003	0.029	0.002	0.997	0.034	0.003	0.019	0.002
0.010	0.001	0.007	0.001	0.998	0.009	0.002	0.006	0.001
0.010	0.001	0.007	0.001	0.999	0.010	0.001	0.007	0.001
0.010	0.001	0.007	0.000	0.999	0.011	0.001	0.009	0.001
0.000	0.000	0.000	0.000	0.999	0.000	0.001	0.000	0.000
0.010	0.001	0.007	0.000	1.000	0.012	0.001	0.012	0.001
0.000	0.000	0.000	0.000	1.000	0.000	0.001	0.000	0.001
0.000		0.000		1.000	0.000	0.000	0.000	0.000
$\Sigma(C(t)\mathrm{d}t) = 1.4$		$\Sigma(E(t)\mathrm{d}t) = 1.0$			$t_m = 0.38$ min			$\sigma^2 = 0.029$ min^2





(b) For the dispersion model to be applied, first we should assume $\text{Bo} < 0.01$ and then:

$$\sigma^2 = 2 \cdot \text{Bo}$$

We obtain $\text{Bo} = 0.014$, so this assumption is almost valid. We can do now:

$$\text{Pe} = \frac{1}{\text{Bo}} = 69.05$$

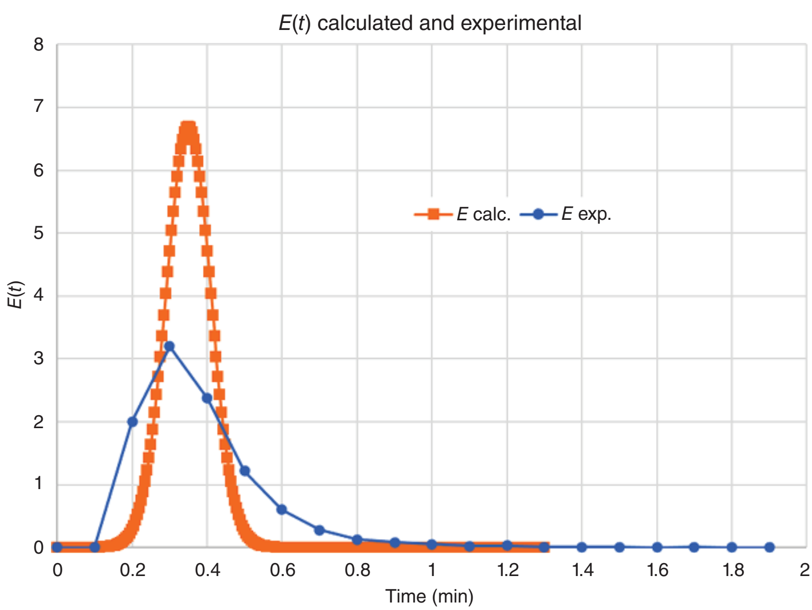
(c) With the values of t_m and σ^2, using the TIS model:

$$n_t = \frac{t_m^2}{\sigma^2} = \frac{0.38^2}{0.029} = 4.98 \text{ tanks} \approx 5 \text{ tanks}$$

(d) This reactor has a very small dispersion, and so the number of tanks that should be used in the TIS model is quite high. The system is completely solvable and its RTD is:

$$E = \frac{1}{\bar{t} \cdot \sqrt{4 \cdot \pi \cdot \text{Bo}}} \cdot \exp\left[-\frac{\left(1 - \frac{t}{\bar{t}}\right)^2}{4 \cdot \text{Bo}}\right]$$

We can compare the experimental and estimated $E(t)$ curves using the spreadsheet:



Problem 1.21 A common issue encountered in the development of gas and liquid phase chromatographic analysis methods is the phenomenon of "tailing." Tailing makes it challenging to characterize the retention time of a specific species due to the prolonged tail associated with the peak. An approximate representation of tailing

can be described by the time dependence of the effluent concentration of the species as follows:

$$C_A = 0 \qquad \text{for } 0 < t < 8.5 \text{ min}$$

$$C_A = 10^5\, e^{-2t} \qquad \text{for } t > 8.5 \text{ min}$$

for C_A in pmol/ml. The curve is generated by injecting a pulse of a sample containing species A into a solvent flowing through the chromatographic column at a constant flow rate.

(a) Determine the mathematical form of the $F(t)$ curve corresponding to the response of the pulse test.
(b) Use the $F(t)$ curve to calculate the average residence time of species A in the column.

Solution to Problem 1.21

For details refer the Wiley website at http://www.wiley-vch.de/ISBN9783527354115

(a) The total area of tracer is:

$$\int_0^\infty C(t)\mathrm{d}t = \int_0^\infty 10^5 \cdot \exp(-2t)\mathrm{d}t = 2.01 \cdot 10^{-3}$$

So, for $t > 8.5$:

$$E(t) = \frac{C(t)}{\int_0^\infty C(t)\mathrm{d}t} = 4.83 \cdot 10^7 \cdot \exp(-2t)$$

We can check that:

$$\int_0^\infty E(t)\mathrm{d}t = 1$$

In this way:

$$F(t) = \int_0^t E(t)\mathrm{d}t = \int_0^t (4.83 \cdot 10^7 \cdot \exp(-2t))\mathrm{d}t = 1 - 2.415 \cdot 10^7 \exp(-2t)$$

(b)

$$t_m = \int_0^\infty t \cdot E(t)\mathrm{d}t = \int_{8.5}^\infty t \cdot (4.83 \cdot 10^7 \cdot \exp(-2t))\mathrm{d}t = 8.89 \text{ min}$$

We can use a numerical method too. In the spreadsheet corresponding to this problem, values of time are given and C_A is calculated. Following the procedure shown before in other problems:

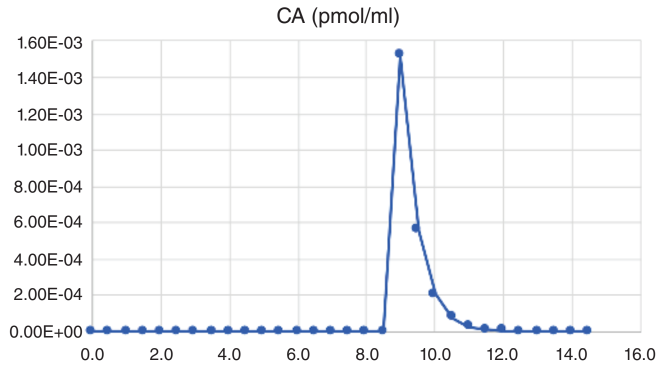
$$\Sigma(C(t)\mathrm{d}t) = 0.001\,204\,66$$

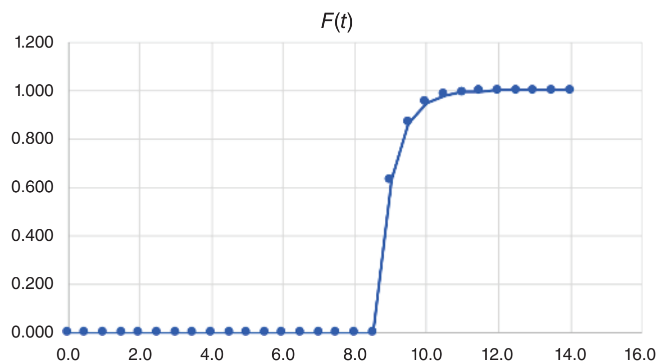
$$\Sigma(E(t)\mathrm{d}t) = 1.0$$

$$t_m = 9.29 \text{ min}$$

$$\sigma^2 = 0.229 \text{ min}^2$$

Doing the graphical plot:





Problem 1.22 The techniques discussed in this chapter for analyzing residence time distribution functions can be applied to study flow conditions in streams or rivers, particularly when assessing the dispersion of pollutants from a source. The following data was obtained from a study of the South Platte River. The average flow rate is 15.68 m^3/s, the length of the reach is 6065 m, and the natural concentration of K^+ ions in the stream is 8.2 mg/l.

At time zero, 453.5 kg of K_2CO_3 was dumped into the upstream end of the reach. Samples were periodically collected at the downstream end of the reach and analyzed for K^+ ions. The results are presented in the table below.

t (min)	K^+ at downstream (g/m^3)
0.0	8.2
60.0	8.2
75.0	8.2
90.0	8.2
105.0	8.4
120.0	9.6
130.0	13.6

t (min)	K^+ at downstream (g/m³)
132.5	14.8
134.0	14.8
138.0	14.6
142.5	13.2
150.0	12.8
165.0	10
180.0	9.2
195.0	8.2
210.0	8.2
300.0	8.2

(a) Calculate the fraction of the tracer (K_2CO_3) that was recovered.
(b) Based on the recovered amount of tracer, calculate the $F(t)$ curve. Additionally, compute the average residence time and plot the effluent concentration of K^+ ions exceeding the background level.

Solution to Problem 1.22

For details refer the Wiley website at http://www.wiley-vch.de/ISBN9783527354115

(a) Subtracting the natural K^+ level, we find:

t (min)	*C*(*t*)	*C*(*t*)·Δ*t*	*E*(*t*)	*E*(*t*)·Δ*t*	*F*(*t*)	*t*·*E*(*t*)·Δ*t*	$(t - t_m)^2 \cdot E(t) \cdot \Delta t$
0.0	0	0.00	0.000	0.000	0.000	0.000	0.000
60.0	0	0.00	0.000	0.000	0.000	0.000	0.000
75.0	0	0.00	0.000	0.000	0.000	0.000	0.000
90.0	0	1.50	0.000	0.006	0.000	0.000	0.000
105.0	0.2	12.00	0.001	0.051	0.013	0.666	9.368
120.0	1.4	34.00	0.006	0.144	0.101	5.992	33.747
130.0	5.4	15.00	0.023	0.063	0.330	18.389	36.857
132.5	6.6	9.90	0.028	0.042	0.399	8.331	9.322
134.0	6.6	26.00	0.028	0.110	0.441	5.577	4.366
138.0	6.4	25.65	0.027	0.108	0.550	14.945	6.567
142.5	5.0	36.00	0.021	0.152	0.645	15.178	1.840
150.0	4.6	48.00	0.019	0.203	0.791	22.234	3.212
165.0	1.8	21.00	0.008	0.089	0.905	31.294	32.824
180.0	1	7.50	0.004	0.032	0.968	15.124	68.928
195.0	0	0.00	0.000	0.000	0.968	5.707	42.389
210.0	0	0.00	0.000	0.000	0.968	0.000	0.000
300.0	0		0.000		0.968	0.000	0.000

$$\Sigma(C(t)\mathrm{d}t) = 236.55$$

$$\Sigma(E(t)\mathrm{d}t) = 1.0$$

$$t_m = 143.44\ \mathrm{min}$$

$$\sigma^2 = 249.42\ \mathrm{min}^2$$

We can see that:

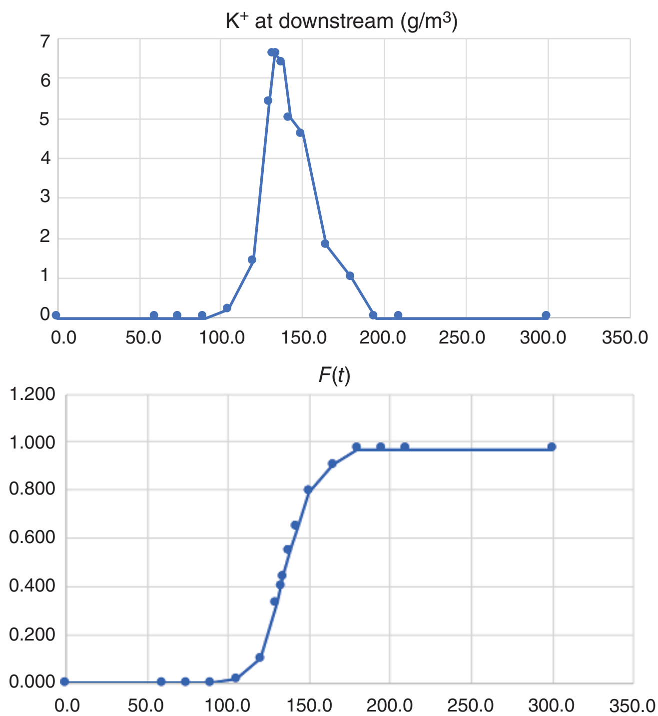
$$\sum(C(t)\mathrm{d}t) = 236.55\ \frac{\mathrm{g}\cdot\mathrm{min}}{\mathrm{m}^3}$$

Taking into account that the flow rate is 15.68 m^3/s, we have that the amount recovered is:

$$M = 236.55\ \mathrm{g}\cdot\frac{\mathrm{min}}{\mathrm{m}^3}\cdot 15.68\ \frac{\mathrm{m}^3}{\mathrm{s}}\cdot 60\ \frac{\mathrm{s}}{\mathrm{min}} = 222\,546.24\ \mathrm{g} = 222.5\ \mathrm{kg}$$

This means that only 49% of the K^+ dumped is recovered.

(b)



Problem 1.23 By injecting brine into an experimental continuous reactor, a signal is obtained at the exit given by:

t (min)	1	3	5	7	9	11	13
C (g/l)	0	2	3	4	2	1	0

The flow rate at the inlet is 10 l/min.

(a) How much salt was injected?
(b) Calculate $E(t)$, $F(t)$, mean residence time, and variance.

Solution to Problem 1.23
For details refer the Wiley website at http://www.wiley-vch.de/ISBN9783527354115

To solve this problem, we should use a spreadsheet. First, data given in the statement is introduced, and then we should do the following calculations:

t (min)	$C(t)$	$C(t)\cdot\Delta t$	$E(t)$	$F(t)$	$t\cdot E(t)\cdot\Delta t$	$(t-t_m)^2\cdot E(t)$	$(t-t_m)^2\cdot E(t)\cdot\Delta t$
1	0.00	0.00	0.0000	0.0000	0	0	0
3	2.00	4.00	0.0833	0.1667	0.5000	1.0208	2.0417
5	3.00	6.00	0.1250	0.4167	1.2500	0.2813	0.5625
7	4.00	8.00	0.1667	0.7500	2.3333	0.0417	0.0833
9	2.00	4.00	0.0833	0.9167	1.5000	0.5208	1.0417
11	1.00	2.00	0.0417	1.0000	0.9167	0.8438	1.6875
13	0.00	0.00	0.0000	1.0000	0.0000	0.0000	0.0000
	$\Sigma(C(t)\mathrm{d}t) = 24.00$				$t_m = 6.50$ min		$\sigma^2 = 5.42$ min^2

The total area of the $C(t)$ curve is:

$$\text{Area} = \int_0^\infty C(t)\mathrm{d}t = \sum_{\text{all points}} C(t)\Delta t = 24\ \frac{\mathrm{g}}{\mathrm{l}}\cdot \mathrm{min}$$

The total amount of tracer introduced is:

$$M_0 = Q_0 \int_0^\infty C(t)\mathrm{d}t = 10\ \frac{\mathrm{l}}{\mathrm{min}} \cdot 24\ \frac{\mathrm{g}\cdot\mathrm{min}}{\mathrm{l}} = 240\ \mathrm{g}$$

For calculating the RTD:

$$E(t) = \frac{C(t)}{\int_0^\infty C(t)\cdot dt} = \frac{C(t)}{24}$$

And:

$$t_m = \int_0^\infty t\cdot E(t)\mathrm{d}t = \sum_{\text{all points}} t\cdot E(t)\cdot \Delta t = 6.5\ \mathrm{min}$$

In a similar way, variance is calculated:

$$\sigma^2 = \sum_{\text{all points}} (t-t_m)^2 \cdot E(t)\cdot \Delta t = 5.41\ \mathrm{min}^2$$

Finally, we can calculate $F(t)$ by attending to its definition:

$$F(t) = \sum_{\text{all points at time}<t} E(t)\cdot \Delta t$$

We can check the form of the graphs showing the distributions: