

1

Viscoelasticity

1.1 Introduction

Thermosetting resins used as the matrix of fiber-reinforced polymers (FRPs) show viscoelastic behavior: it is nondestructive time- and temperature-dependent mechanical behavior. Furthermore, the time-temperature superposition principle (TTSP) holds for the viscoelasticity of thermosetting resins. A master curve showing the viscoelasticity of a resin over a wide range of reduced time at a reference temperature can be constructed based on the TTSP. In addition, the long-term nondestructive mechanical behavior of the resin can be predicted from this master curve.

It can be readily inferred that FRPs also show viscoelastic behavior as a result of the influence of the viscoelasticity of the matrix resin, and that the same TTSP as for the matrix resin also holds for FRP viscoelasticity. Furthermore, it can be inferred that the same TTSP as that for matrix resin holds for the static, creep, and fatigue strengths of FRP, and that master curves of these strengths can be constructed to predict the long-term degradation of these strengths.

1.2 Concept of Viscoelastic Behavior

The concept of linear viscoelastic behavior is explained using a Maxwell model with a spring and dashpot, as shown in Figure 1.1. The spring is a solid having an elastic

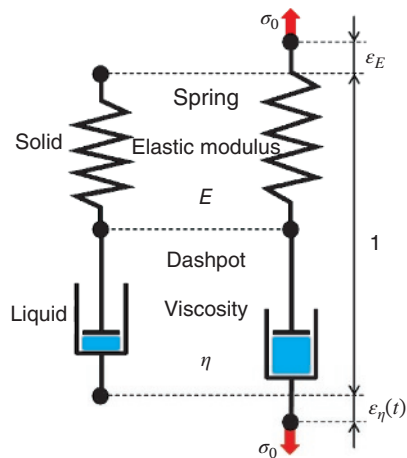


Figure 1.1 Maxwell model.

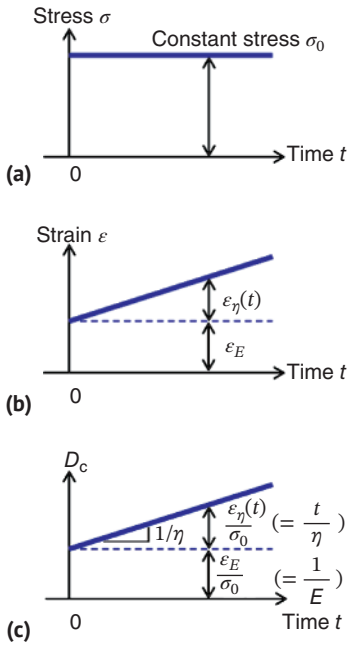


Figure 1.2 Creep compliance of the Maxwell model.

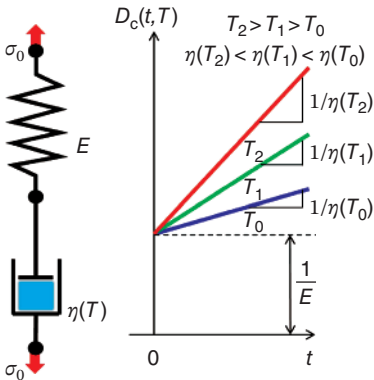


Figure 1.3 Creep compliance of the Maxwell model at various temperatures.

modulus E . The dashpot is a liquid having viscosity η . When this Maxwell model is loaded using a constant stress σ_0 , as shown in Figure 1.2a, the total strain $\epsilon(t) = \epsilon_E + \epsilon_\eta(t)$ is generated, as shown in Figure 1.2b. Creep compliance $D_c(t)$ shown in Figure 1.2c is definable by the following equation:

$$D_c(t) = \frac{\epsilon(t)}{\sigma_0} = \frac{\epsilon_E}{\sigma_0} + \frac{\epsilon_\eta}{\sigma_0} = \frac{1}{E} + \frac{t}{\eta} \quad (1.1)$$

1.3 Concept of TTSP

Although the elastic modulus E of the spring in the Maxwell model does not change with temperature, the viscosity η of dashpot decreases drastically with increasing temperature, as shown in Figure 1.3. Figure 1.4 presents the creep compliance of the Maxwell model with various temperatures against time. Each creep compliance keeps a constant value of $1/E$ in the short time range, and each maintains a constant slope of $1/\eta$ over a long time range. These creep compliances at various temperatures are superimposed on each other by a horizontal shift, as shown in Figure 1.4. The amount of horizontal shift is defined as the time-temperature shift factor $a_{T_0}(T)$, which is shown by the following equation and Figure 1.5:

$$a_{T_0}(T_i) = \frac{t_i}{t_0}, \quad (1.2)$$

$$\log a_{T_0}(T_i) = \log t_i - \log t_0, \quad (i = 1, 2)$$

The time-temperature shift factor can be regarded as the rate of acceleration by increasing temperature.

1.4 Master Curve of Creep Compliance of Matrix Resin

The procedure to construct a master curve of creep compliance of the matrix resin based on TTSP is presented in this section. First, the creep tests are conducted at various temperatures T_1, T_2 , and T_3 ($T_1 < T_2 < T_3$), as shown in Figure 1.6. The creep compliances D_c against time t at various temperatures are determined by substituting the measured data in Eq. (1.1), as shown on the left side of Figure 1.7.

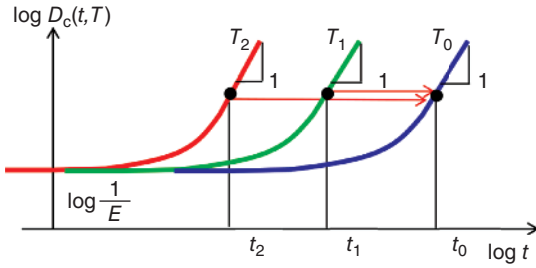


Figure 1.4 Superposition of creep compliances at various temperatures by shifting.

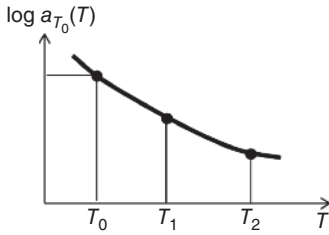


Figure 1.5 Time-temperature shift factor.

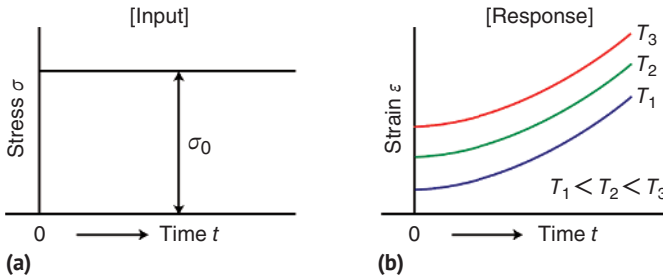


Figure 1.6 Creep tests at various temperatures.

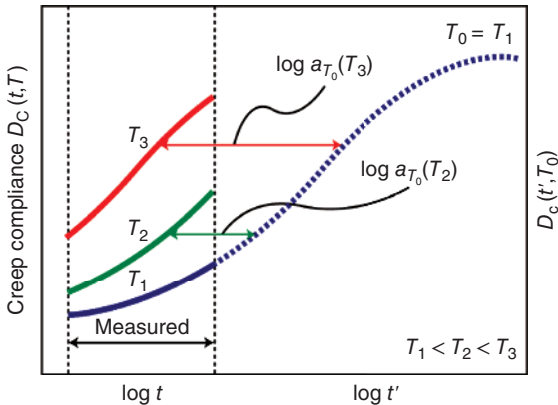


Figure 1.7 Creep compliance at various temperatures and the master curve.

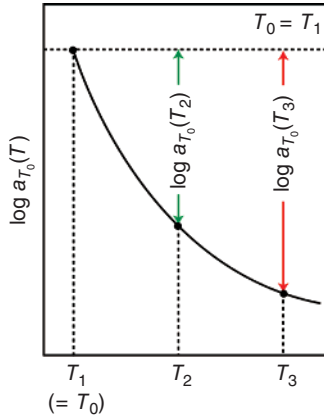


Figure 1.8 Time–temperature shift factor for deformation.

The master curve of creep compliance D_c against the reduced time t' at the reference temperature $T_0 (= T_1)$ is obtainable by superimposing the creep compliances D_c at various temperatures by horizontal shifting, as shown on the right side of Figure 1.7. The long-term creep compliance of the matrix resin can be predicted from this master curve. The amounts of horizontal shift are also obtained as the time–temperature shift factor, as shown in Figure 1.8.

1.5 Generalization of TTSP for Nondestructive Deformation Properties to Static, Creep, and Fatigue Strengths of FRPs

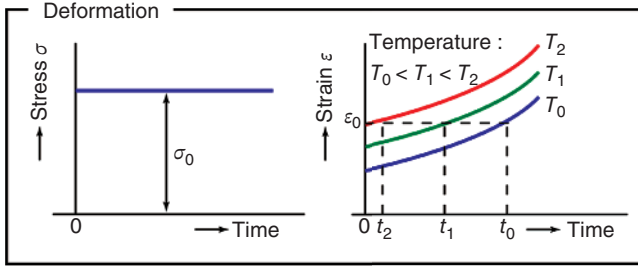
The most important condition for the accelerated testing methodology (ATM) proposed by the authors is the generalization of TTSP for nondestructive deformation properties to the static, creep, and fatigue strengths of FRPs. In this condition, the same TTSP as that used for the deformation of the matrix resin holds for the strengths of FRPs, as shown in Figure 1.9. Concretely, the following equation is used for the time–temperature shift factors:

$$a_{T_0}(T_i) = \frac{t_i}{t_0} = \frac{t_{si}}{t_{s0}} = \frac{t_{ci}}{t_{c0}} = \frac{t_{fi}}{t_{f0}}, \quad (i = 1, 2, \dots) \quad (1.3)$$

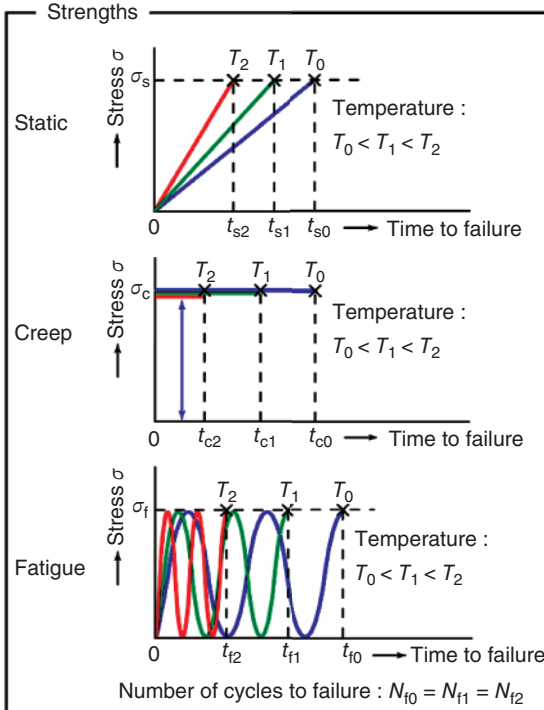
Therefore, the long-term strengths of FRPs can be predicted from their measured short-term strengths at elevated temperatures and from the time–temperature shift factor $a_{T_0}(T)$ for the deformation of the matrix resin.

1.6 Master Curve of Static Strength of FRP

The procedure to construct a master curve of static strength of an FRP using the time–temperature shift factor for the matrix resin deformation is presented in this section. First, the static tests are conducted at various temperatures T and strain



(a)



(b)

Figure 1.9 Generalization of TTSP for nondestructive deformation properties to static, creep, and fatigue strengths of FRP.

rates R , as shown in Figure 1.10a,b. The static strengths σ_s against the failure time t_s at various temperatures are determined from the measured data, as shown on the left side of Figure 1.10c, where the failure time t_s is defined as the period from the beginning of loading to the failure load.

The master curve of static strength σ_s against the reduced failure time t'_s at reference temperature T_0 is obtainable by horizontally shifting of the static strength σ_s at various temperatures, as shown on the right side of Figure 1.10c. The long-term static strength of the FRP can be predicted from this master curve. The amount of horizontal shift is the time-temperature shift factor for the matrix resin deformation.

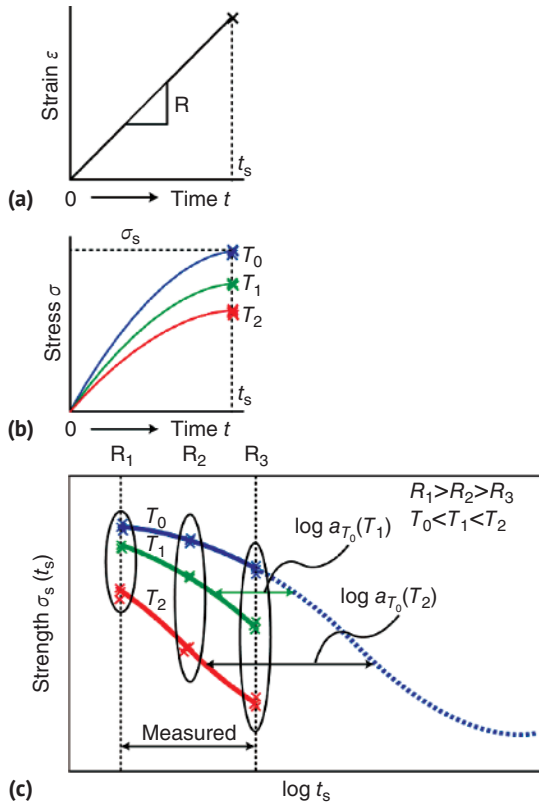


Figure 1.10 How to construct the master curve of static strength.

1.7 Master Curve of Creep Strength of FRP

The procedure to construct the master curve of creep strength of FRPs using the time-temperature shift factor for the deformation of the matrix resin is shown in this section. First, the creep tests are conducted at various temperatures T and constant stresses σ_c , as shown in Figure 1.11a. The creep strength σ_c against failure time t_c at various temperatures is determined from measured data, as shown on the left side of Figure 1.11b.

The master curve of creep strength σ_c against reduced failure time t'_c at the reference temperature T_0 is obtainable by shifting the creep strength σ_c horizontally at various temperatures, as shown on the right side of Figure 1.11b. The long-term creep strength of the FRP can be predicted from this master curve. The amount of horizontal shift is the time-temperature shift factor for the matrix resin deformation.

1.8 Master Curve of Fatigue Strength of FRP

The procedure used to construct the master curve of fatigue strength of FRPs using the time-temperature shift factor for the matrix resin deformation is shown in this section. First, the fatigue tests are conducted at various temperatures T and various

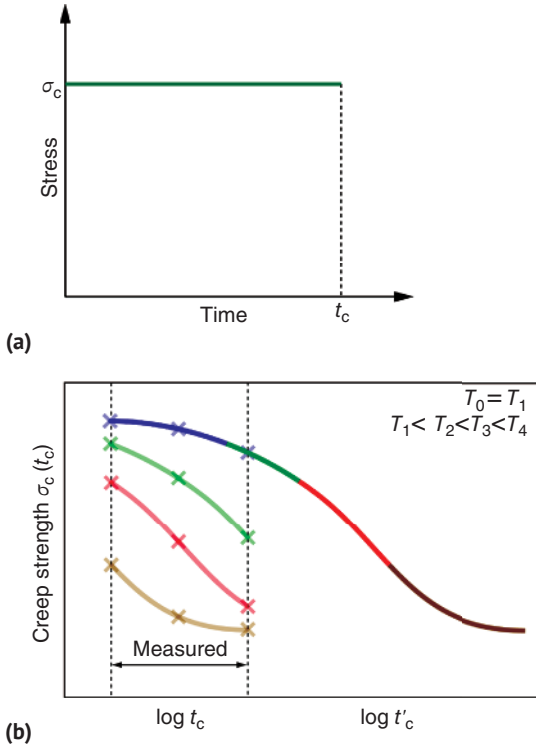


Figure 1.11 How to construct the master curve of creep strength.

maximum stresses σ_f under a constant frequency f_0 , as shown in Figure 1.12a. The fatigue strengths σ_f against the failure time t_f at various temperatures under a constant frequency f_0 are determined from the measured data, as shown on the left side of Figure 1.12b. The master curve of static strength in this figure can be regarded as the fatigue strength at the number of cycles to failure $N_f = 1/2$.

The fatigue strength σ_f against the reduced failure time t'_f at the reference temperature T_0 under various corresponding frequencies f_i is obtainable by shifting the fatigue strength σ_f horizontally at various temperatures, as shown on the right side of Figure 1.12c. The corresponding frequency f_i for T_i is obtained as shown in the following equation:

$$\frac{f_0}{f_i} = a_{T_0}(T_i), \quad (i = 1, 2, \dots) \tag{1.4}$$

The long-term fatigue strength σ_f against the reduced failure time t'_f at the reference temperature T_0 for an arbitrary frequency f can be predicted from this figure. The amount of horizontal shift is the time–temperature shift factor for matrix resin deformation.

The master curves of fatigue strength σ_f at various numbers of cycles to failure N_{fi} are obtained by connecting the fatigue strength σ_f at various frequencies f_i at

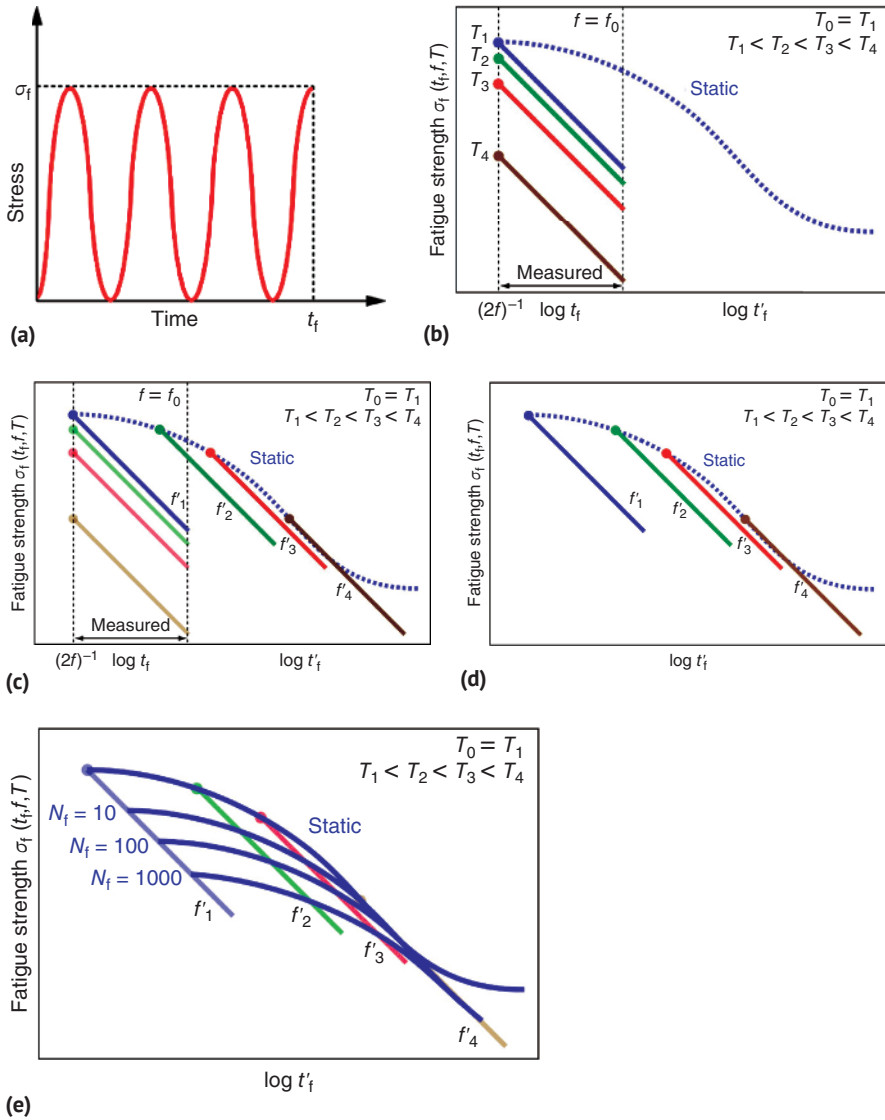


Figure 1.12 How to construct the master curve of fatigue strength.

the same number of cycles to failure N_{fi} , as shown in Figure 1.12d,e. The long-term fatigue strength σ_f against the reduced failure time t'_f at reference temperature T_0 for an arbitrary number of cycles to failure N_f can be predicted from Figure 1.12e.

1.9 Conclusion

The concepts of viscoelasticity and TTSP were explained using the Maxwell model. The generalization of TTSP for nondestructive deformation properties of the matrix resin to static, creep, and fatigue strengths of FRPs was introduced as the most

important condition for the ATM. Procedures to obtain the master curves of static, creep, and fatigue strengths of FRPs were explained. Readers who want more details related to viscoelasticity may refer to [1].

Reference

- 1 Christensen, R.M. (2003) *Theory of Viscoelasticity*, 2nd edn, Dover Publications, Inc., Mineola, New York.

